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Applied Mathematics 10

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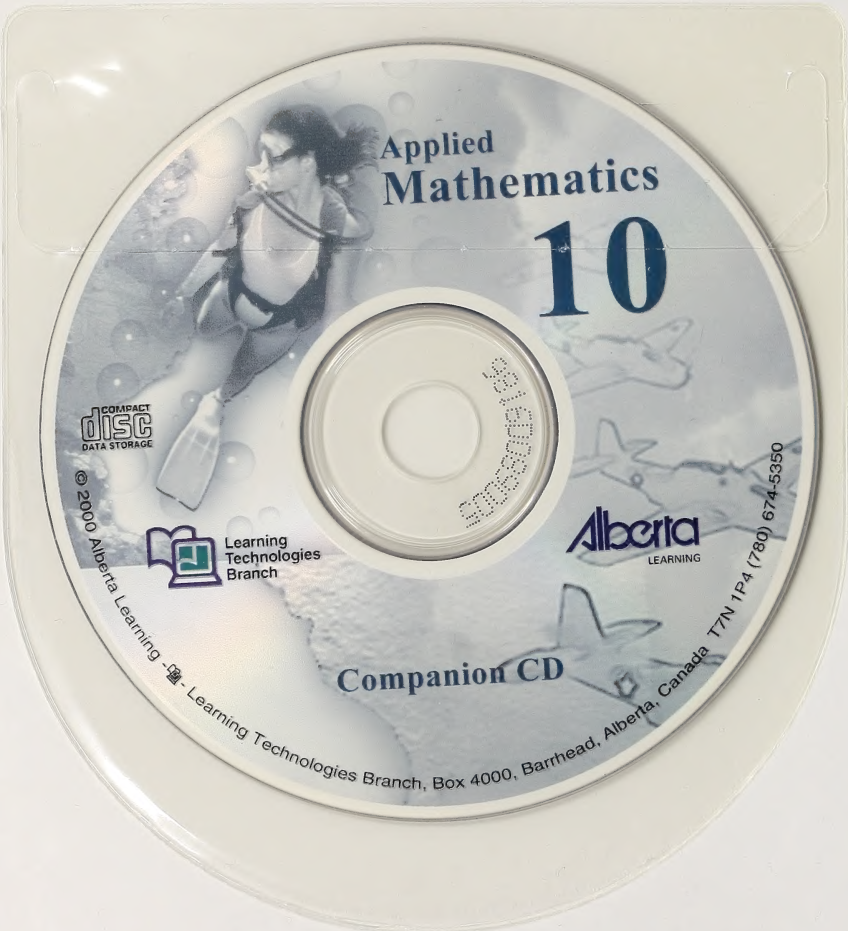

Applied Mathematics 10

Companion CD

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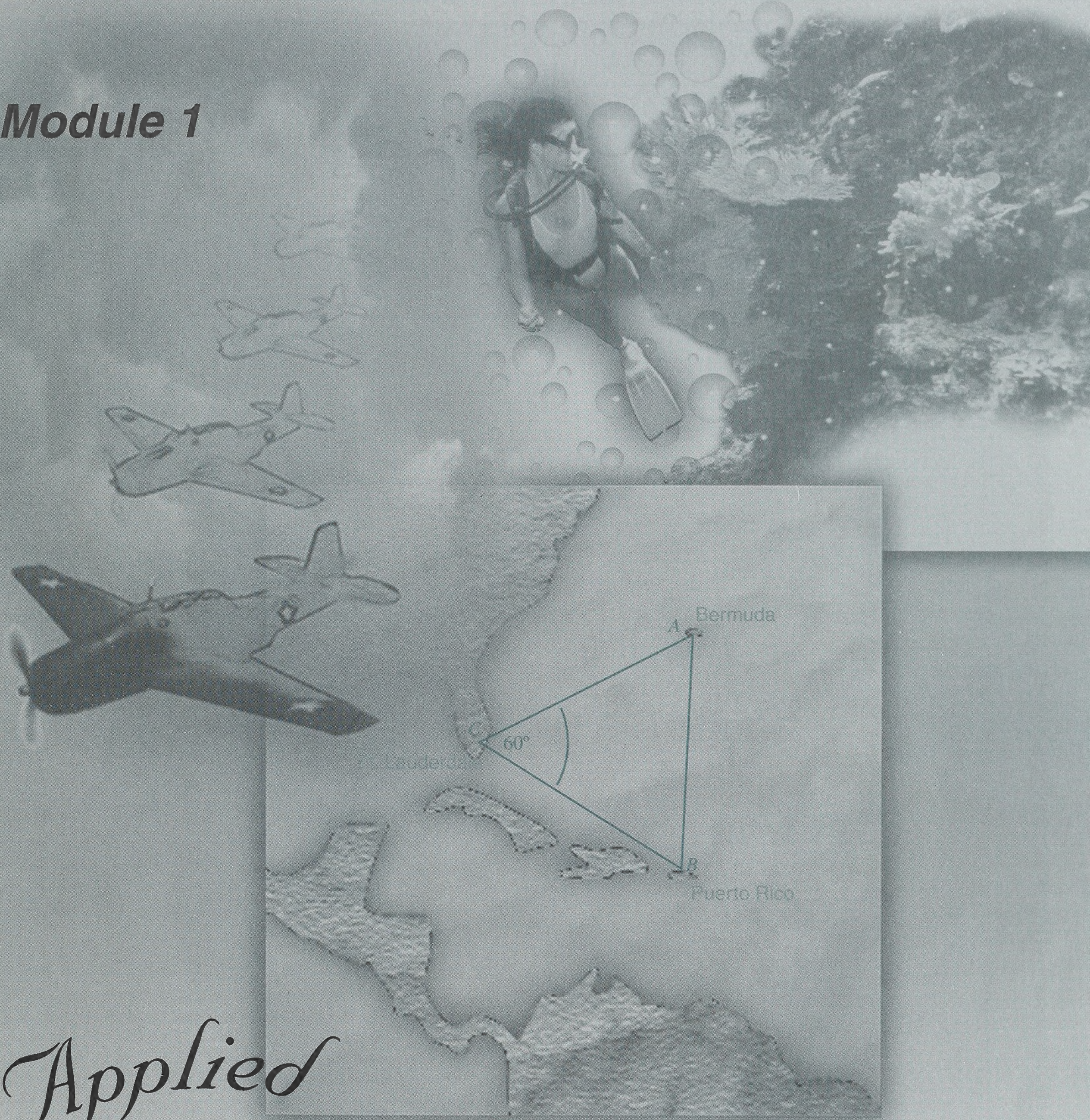
Module 1



Applied Mathematics 10

MEASUREMENT

Learning Technologies Branch

Alberta LEARNING

Applied Mathematics 10
Student Module Booklet
Module 1
Measurement
Learning Technologies Branch
ISBN 0-7741-1607-2

Cover photo: COREL CORPORATION

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

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Welcome

Applied Mathematics 10

Module 1
Measurement

Module 2
Number Patterns in Tables

Module 3
Relations and Functions

Module 4
Sampling

Module 5
Line Segments

Module 6
Linear Functions

Module 7
Trigonometry

Welcome to Module 1. We hope you'll enjoy your study of Measurement.

Applied Mathematics 10 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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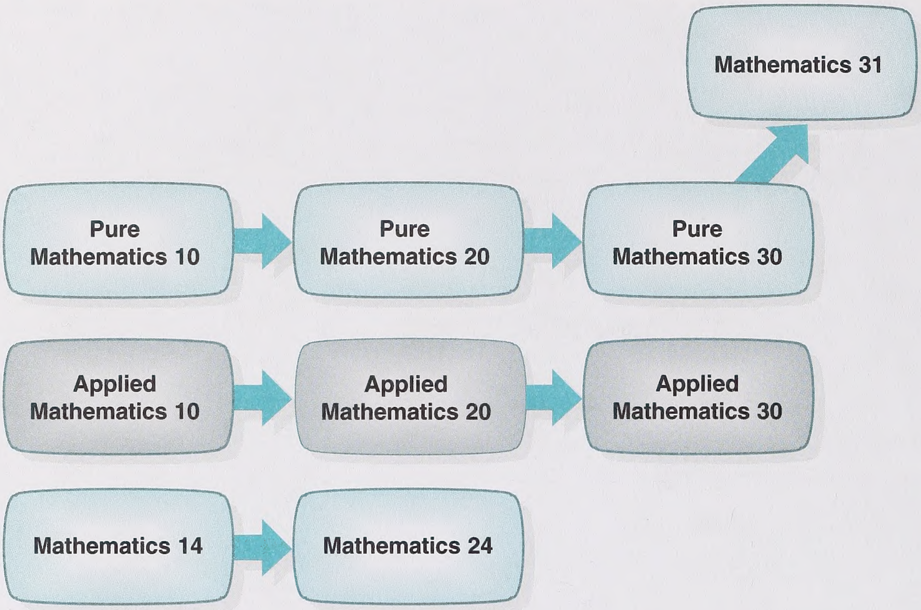
Introduction to Applied Mathematics 10

Applied Mathematics 10 is the first course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different post-secondary and career plans.

You may find it helpful to read mathematics updates on Alberta Learning's website:

http://www.learning.gov.ab.ca/studentprograms

Before enrolling in Applied Mathematics 10, it is recommended that you talk with a school counsellor about your career plans.

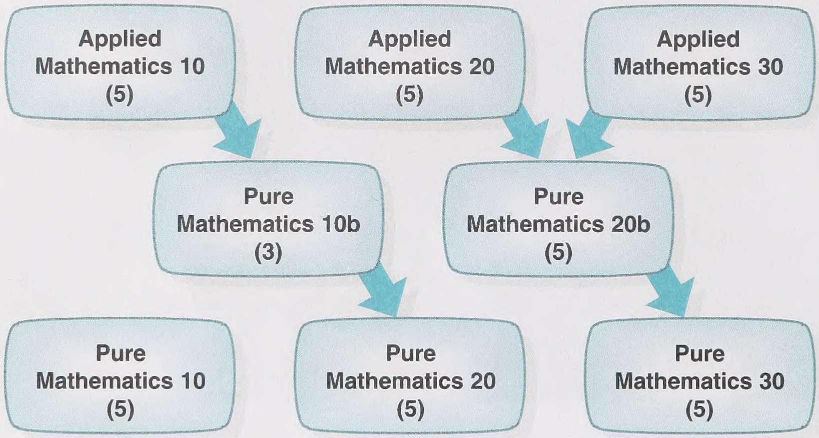
TRANSFERRING FROM THE APPLIED MATHEMATICS PROGRAM

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

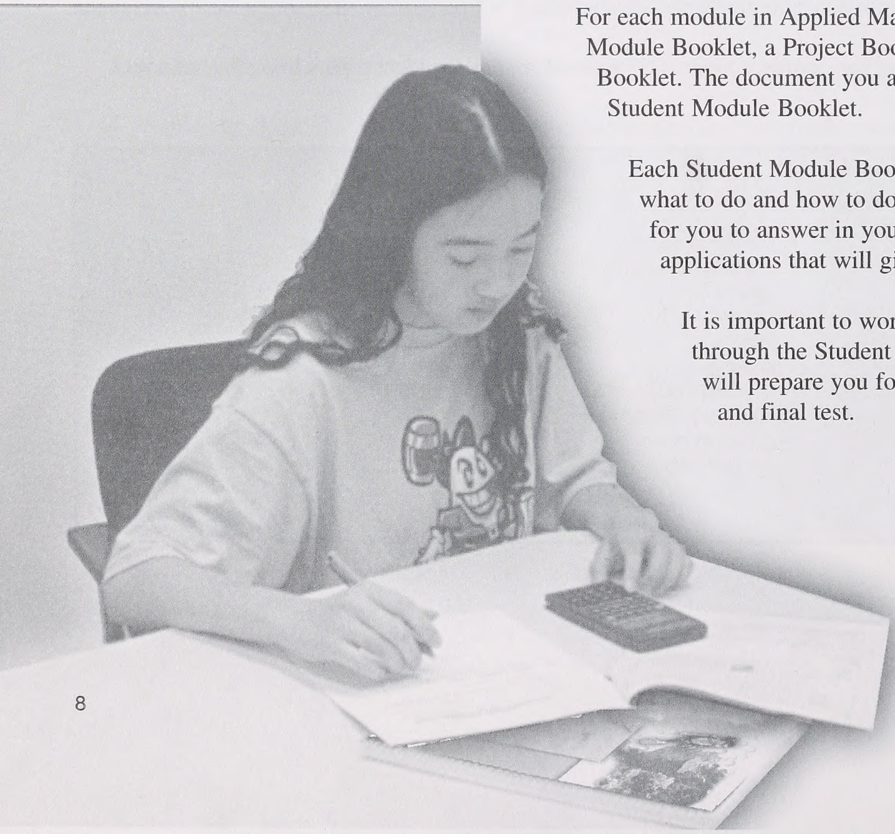
The following table shows some of the common topics and some of the independent topics in the mathematics program.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may take the 3-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may take the 5-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.



STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 10



For each module in Applied Mathematics 10, there is a Student Module Booklet, a Project Booklet, and an Assignment Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the projects, assignments, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also, store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words, and recording useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience) or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 10 to be your study partner. You will find that having a friend with whom you can discuss mathematical ideas will make your studying more enjoyable.

Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 10, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 10 Source Book,* Western Canadian Edition, published by Addison Wesley Longman Ltd. (1999)

- a binder, lined loose-leaf paper, graph paper, dividers, a pencil, and an eraser

- metric and imperial measuring devices, such as a ruler, a tape measure, a yardstick, a vernier caliper, and a micrometer

- a mathematical instrument set (compass, protractor, and triangles)

- a computer installed with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and *Microsoft® Excel*. The examples in this course show *Microsoft® Excel.*

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Brands	Texas Instruments	Sharp	Casio
Models	TI-82	EL-9600c	CFX-9850Ga-Plus
	TI-83	EL-9600*	CFX-9850G*
	TI-83 Plus	EL-9300*	CFX-9800G*
	TI-86	EL-9200*	FX-9700 series*
	TI-89		
	TI-92		
	TI-92 Plus		

*no longer commercially available, but may be available on loan from your school division

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you view the video *The TI-83 Graphing Calculator Video Tutor* to discover some of the calculator's features.

Many of the resources you will need for this course may be purchased from the Learning Resources Distributing Centre (LRDC). Following is the LRDC website:

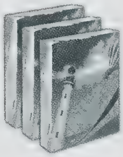
http://www.lrdc.edc.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

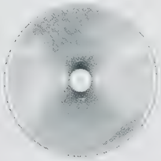
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the explanations given to discover what the various icons prompt you to do.

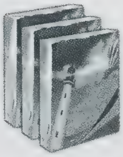
- Refer to the textbook.



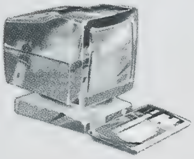
- Use the companion CD for Applied Mathematics 10.



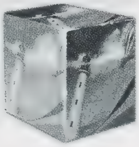
- Use mathematical instruments, measuring devices, and other materials.



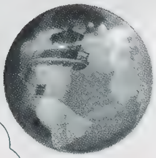
- Work with a computer.



- Complete the module project or assignment.

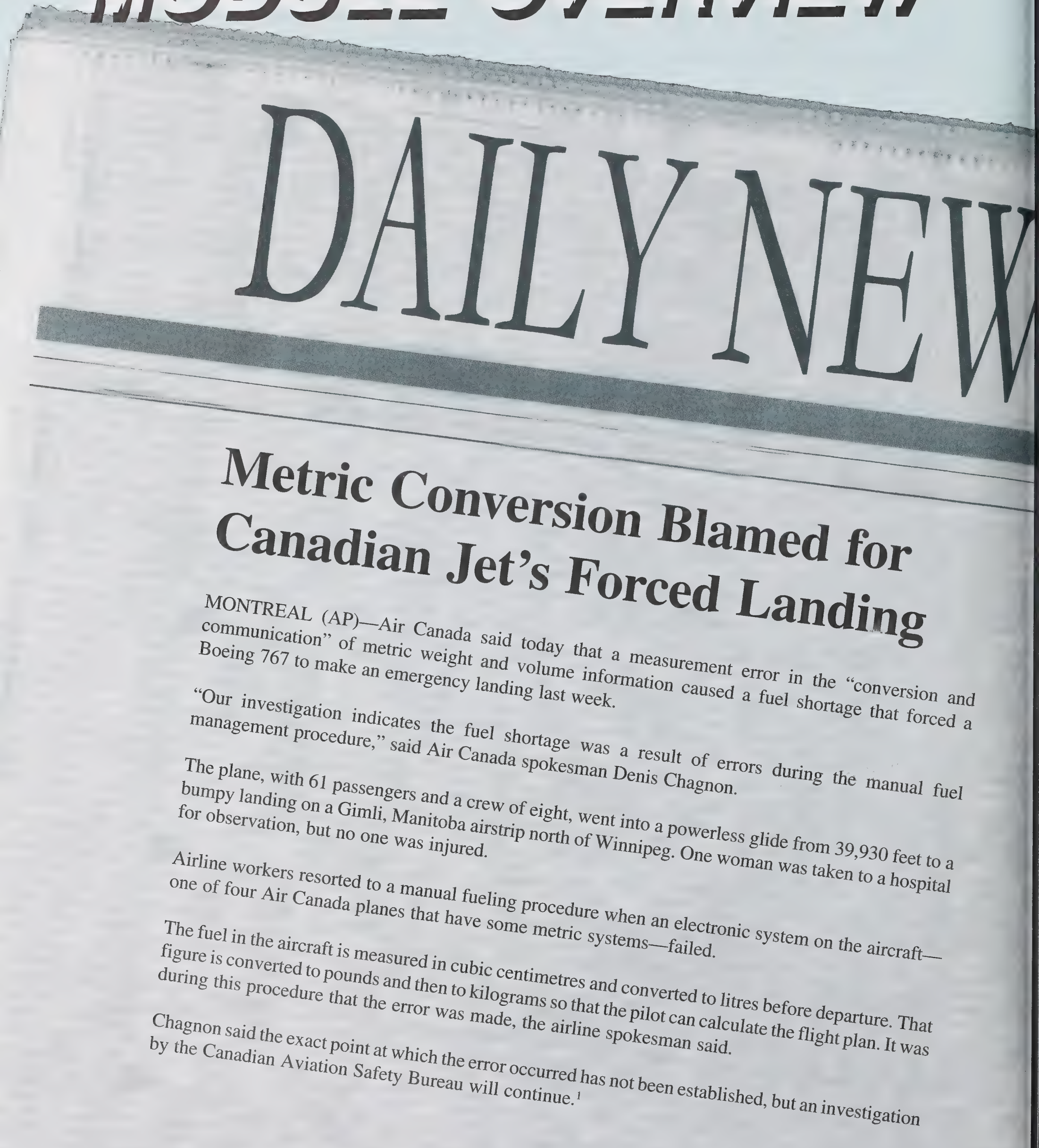


- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW



Metric Conversion Blamed for Canadian Jet's Forced Landing

MONTREAL (AP)—Air Canada said today that a measurement error in the "conversion and communication" of metric weight and volume information caused a fuel shortage that forced a Boeing 767 to make an emergency landing last week.

"Our investigation indicates the fuel shortage was a result of errors during the manual fuel management procedure," said Air Canada spokesman Denis Chagnon.

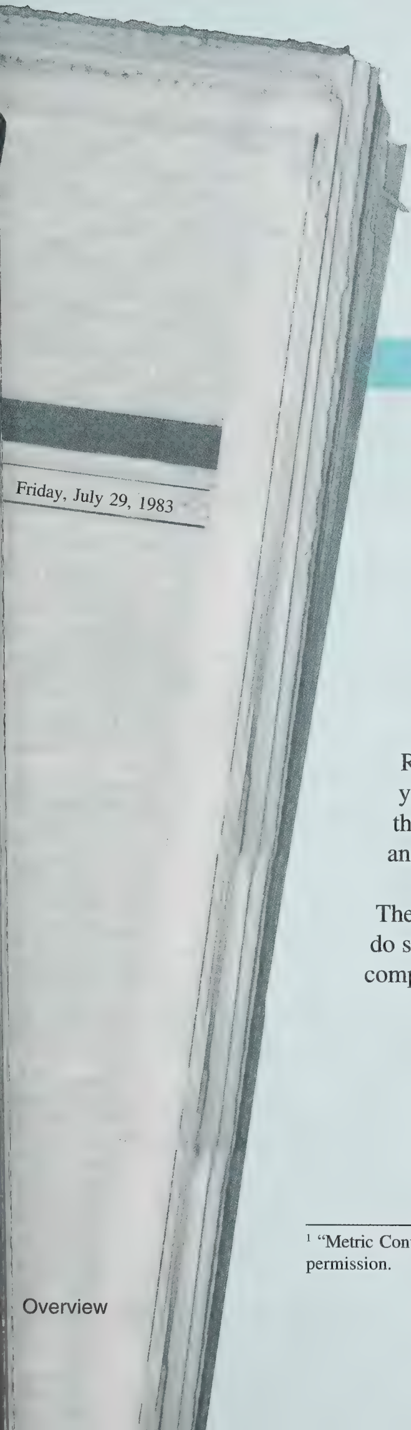
The plane, with 61 passengers and a crew of eight, went into a powerless glide from 39,930 feet to a bumpy landing on a Gimli, Manitoba airstrip north of Winnipeg. One woman was taken to a hospital for observation, but no one was injured.

Airline workers resorted to a manual fueling procedure when an electronic system on the aircraft—one of four Air Canada planes that have some metric systems—failed.

The fuel in the aircraft is measured in cubic centimetres and converted to litres before departure. That figure is converted to pounds and then to kilograms so that the pilot can calculate the flight plan. It was during this procedure that the error was made, the airline spokesman said.

Chagnon said the exact point at which the error occurred has not been established, but an investigation by the Canadian Aviation Safety Bureau will continue.[1]

It may seem difficult to believe that a passenger jet could run out of gas because the ground crew made a measurement error. However, this incident actually happened, and a movie was made about it. The movie was called *Falling from the Sky: Flight 174*, and it was released in 1995.



The story of Flight 174 illustrates why taking and estimating measurements is a very important skill. Making errors can have costly results.

In this module, you will use measuring devices to take direct measurements. You will make estimates and calculate indirect measurements, such as perimeter, area, and volume, and you will solve problems involving measurements.

EVALUATION

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation.

The mark distribution is as follows:

Module Project	**40 marks**
Module Assignment	**60 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 6 in this Student Module Booklet will prepare you for the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

---

[1] "Metric Conversion Blamed for Canadian Jet's Forced Landing," *Associated Press*, 29 July, 1983. Reprinted by permission.

MODULE PROJECT

THE DREAM ROOM

Beginning the Project

Many teenagers dream of having a bedroom that is decorated to suit their interests and personalities. What does your dream room look like? Is it similar to the one in the photograph?

Your module project for Module 1, Measurement, is the Dream Room. This project involves planning and researching the costs of redecorating a bedroom with an attached bathroom.

Because this is a dream room, you can pretend the room is any size you want it to be. However, you will be expected to measure real doors, windows, walls, and furniture, so that your measurements are realistic.

You will be working with a budget of $5000. You will have to research the cost for new baseboards, floor coverings, paint or wallpaper, and window treatments. You will also be expected to include the prices for a new bed, dresser, desk, chair, and lamp. You can include any other items that will fit into your budget. You can assume that the bathroom already has the fixtures you want, and you do not have to consider any installation costs.

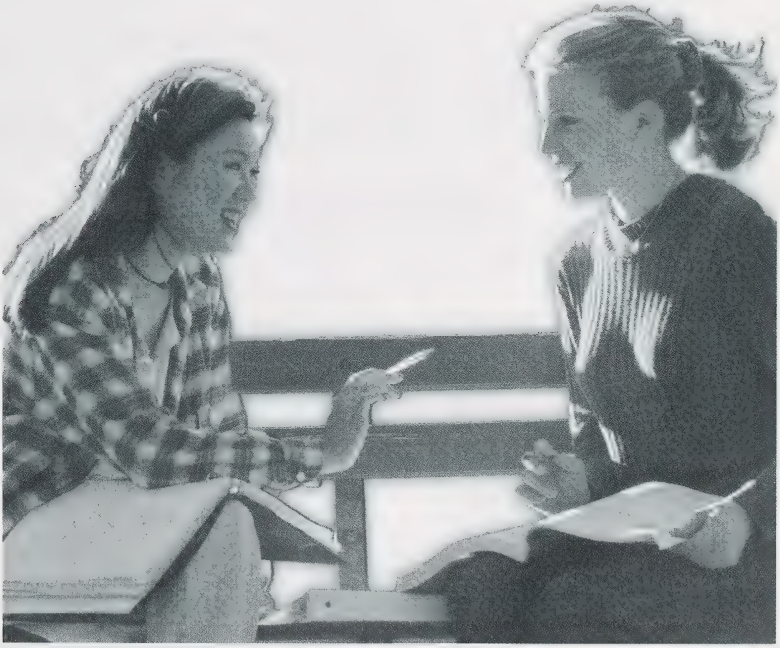
Turn to page 2 of the textbook, read the page, and respond to the questions posed. Store your responses in the project section of your mathematics binder.

After you have recorded some of your initial ideas for your dream room, begin researching decorating ideas and prices. Page 3 of the textbook lists the Internet address of Addison Wesley Longman Ltd. This website has links to several sites you may find helpful.

As you work through Activities 1 to 6, continue to research decorating ideas and prices by visiting stores, reading decorating magazines, looking at catalogues, and surfing the Internet. Talk to family members, friends, and experts in design. Department stores often have personnel ready to assist with many facets of design.

You will find that the imperial system of measurement is often used in everyday world. As you work through Activities 1 to 6, you will learn how to use this system and review how to calculate perimeter and area. These skills will be useful for this project.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner. Just remember that the work on the project you submit must be your own.



ACTIVITY 1

Using Measuring Devices

When you apply for a driver's license, you must state your height and weight (mass).

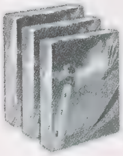
When you give your height in metres or centimetres and your mass in kilograms, you are using the International System of Units (SI), commonly called the **metric system**.

When you give your height in feet and inches and your weight in pounds, you are using the English Customary System of Measurement, commonly called the **imperial system**.

In this activity you will use a variety of metric and imperial measuring devices.

RULERS, TAPE MEASURES, AND TRUNDLE WHEELS

Rulers



A ruler (sometimes called a rule) is a tool for measuring short distances. Rulers include metre-sticks and yardsticks. They can be made of metal, wood, or rigid plastic.

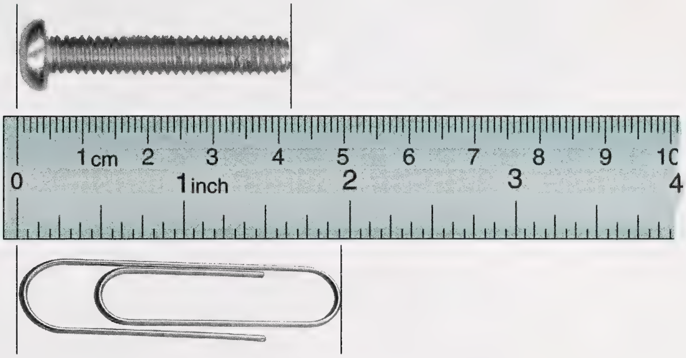
There are two rulers and a metre-stick in the photograph on page 8 of the textbook.

There is usually a scale along each edge of a ruler, metre-stick, and yardstick. The scale marks divisions in either metric or imperial units. Often one edge of the device has a metric scale and the other edge has an imperial scale. Some metal rulers have scales on the front and back.

Work through the following example to review how to read a ruler.

Example

Use a ruler to measure the objects shown in the following diagram. Give the measure of the bolt in metric units. Give the measure of the paper clip in imperial units.



Solution

This ruler has a metric scale and an imperial scale.

On the metric scale, there are 10 graduation marks per centimetre; therefore, the smallest division of the metric scale is 0.1 cm. The bolt is 4.2 cm.

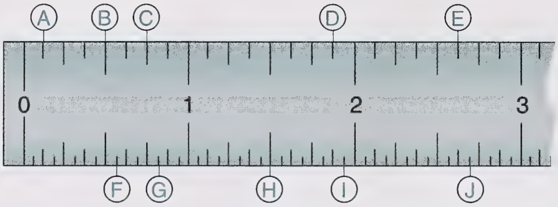
On the imperial scale, there are 16 graduation marks per inch; therefore, the smallest division of the imperial scale is $\frac{1}{16}$ of an inch. The paper clip is $1\frac{15}{16}$ inches.

There are several ways of reporting imperial measurements. For example, $1\frac{15}{16}$ inches may be written as $1\frac{15}{16}$ in. or $1\frac{15}{16}$".

Note: On most rulers, there is a gap between the end of the ruler and the first graduation mark. There is also a gap between the other end of the ruler and the last graduation mark. These gaps allow for wear on the ends of the ruler. When measuring, be sure to align the object and ruler at 0 rather than at the end of the ruler.

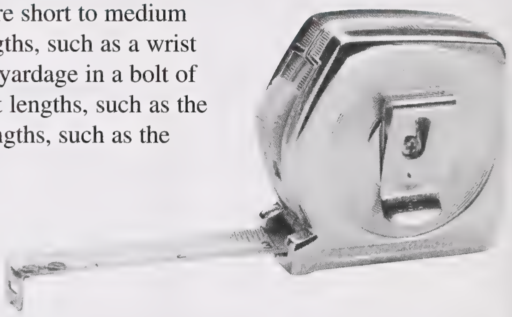
Note, also, that to make the scale easier to read, the graduation marks in each scale are of different lengths.

1. State each measurement illustrated on the following ruler. **Hint:** First determine the number of graduation marks per inch on each scale of the ruler. Give each answer in simplest form.



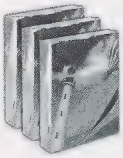
Compare your responses with the suggested answers in the Appendix, Activity 1, page 70.

Tape Measures



A tape measure is a tool used to measure short to medium lengths. Tailors may measure short lengths, such as a wrist measure, or longer lengths, such as the yardage in a bolt of material. Carpenters may measure short lengths, such as the width of a window casing, or longer lengths, such as the length of a house.

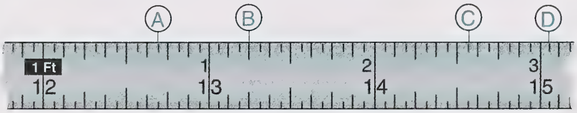
Both types of tape measures must be flexible enough to measure curves—an archway, a waist measurement, and so on. A carpenter's tape measure is usually made of a flexible metal; a tailor's tape measure is made of cloth or a flexible plastic.



There are three kinds of tape measures in the photograph on page 8 of the textbook.

A tape measure, like a ruler, usually has a scale along each edge. Graduation marks on a metric tape measure are in metres and centimetres. Graduation marks on an imperial tape measure are in feet and inches.

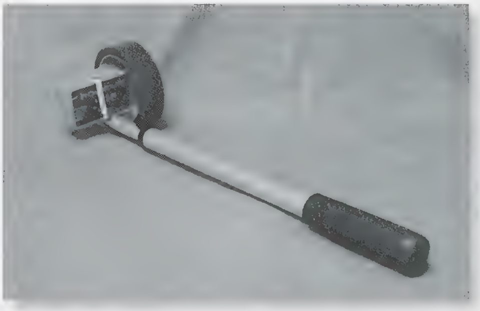
2. State the measurement in feet and inches of each reading illustrated on the following tape measure.



Compare your responses with the suggested answers in the Appendix, Activity 1, page 70.

Trundle Wheels

A trundle wheel (also called a measuring wheel) is a measuring device used to measure long distances. A trundle wheel is rolled along the surface being measured.



ROLATAPE CORPORATION

There is a trundle wheel in the photograph on page 8 of the textbook.

It is believed that ancient Egyptians used trundle wheels when they were constructing the pyramids. Today, trundle wheels are used by construction estimators, police officers, and landscape designers.

Trundle wheels come in a variety of different sizes. For example, there are trundle wheels with diameters of 25 inches, 10 inches, 6 inches, and 4 inches.

The distance a trundle wheel makes in one revolution of the wheel is related to the diameter of the wheel. Take out your graphing calculator and work through the following example.

Example

What distance does a trundle wheel with a diameter of 25 inches cover in one revolution of the wheel? Round the answer to the nearest whole number.

Solution

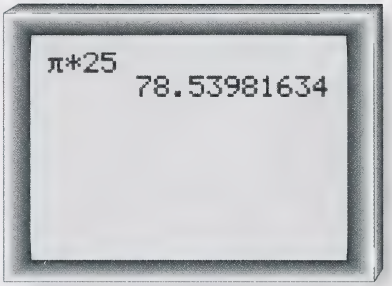
$$\begin{aligned} C &= \pi d \\ &= \pi(25) \\ &\doteq 79 \end{aligned}$$

Press the following key sequence:

2nd [π] × 2 5 ENTER

The brackets indicate a secondary function. This notation means to press the key with π written above it.

For calculations involving π, use the π key on your calculator instead of the rounded value of π.



```
π*25
          78.53981634
```

The trundle wheel covers about 79 inches.

Notice that in the preceding example the key sequence and display for the TI-83 graphing calculator are shown in the solution.

Each of the blue, black, and grey keys on the TI-83 graphing calculator has a primary and secondary function. The primary function is printed in white on the key. The secondary function is printed in yellow above the key. To access any of the secondary functions on your graphing calculator, first press the yellow 2nd key.

3. Calculate the distance each of the following trundle wheels covers in one revolution. Round each answer to the nearest whole number.
    - **a.** a trundle wheel with a diameter of 4 inches
    - **b.** a trundle wheel with a diameter of 6 inches
    - **c.** a trundle wheel with a diameter of 10 inches
4. If a trundle wheel covers 50 cm in one revolution of the wheel, what is the wheel's diameter? Round the answer to nearest whole number.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 70–71.

Gather together the following materials to make a trundle wheel with a circumference of 50 cm: a piece of strong cardboard or firm card, a brass fastener, a fine-line marker or pen, a metric ruler, a compass, a pencil, and scissors.

Next, make a trundle wheel by completing the following steps:

Step 1: Find the radius needed to have a circumference of 50 cm.

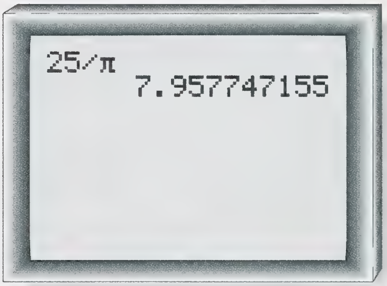
$$C = 2\pi r$$

$$50 = 2\pi r$$

$$\frac{50}{2\pi} = r$$

$$\frac{25}{\pi} = r$$

$$r \doteq 8$$



```
25/π
          7.957747155
```

The trundle wheel needs a radius of approximately 8 cm.

Step 2: Adjust the compass setting so that the pin end of the compass points to 0 cm on a metric ruler and the pencil end points to 8 cm. Lock the radius if you can; otherwise, be careful not to accidentally re-adjust the compass. Use the compass to draw a circle on the cardboard.

Step 3: Carefully cut out the circle. With a fine-line marker or pen make a straight line from the centre of the circle to the edge of the circle. This will help you count the number of revolutions the trundle wheel makes.

Step 4: Cut out a long, narrow strip of cardboard for the handle of the trundle wheel, make a hole at one end, and attach the handle to the circle with a brass fastener.

Practise using the trundle wheel you have made.

If you have to measure the length or width of a room, simply start measuring with the wheel edge touching a wall and roll to other end of the room until the wheel touches the other wall.

Always start measuring by lining the marker line pointed down towards the ground where you want to begin the measurement. Roll the wheel along, counting the number of times the marker line hits the floor. Each revolution covers 50 cm, or 0.5 m.

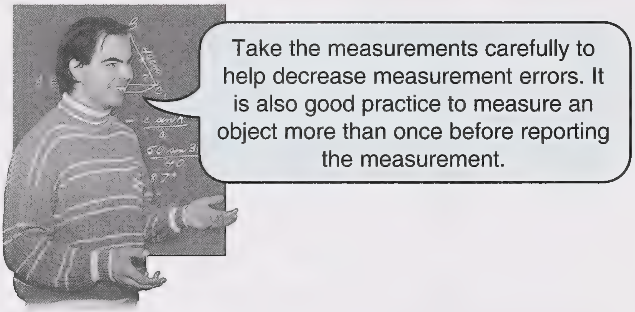
5. On many commercially made trundle wheels, there is a digital display. Why would this feature be helpful when measuring longer distances?

Compare your response with the suggested answer in the Appendix, Activity 1, page 72.

Measuring In and Beyond the Classroom

Now you will apply what you have learned about using rulers, tape measures, and trundle wheels. Gather together these measuring devices.

Turn to page 10 of the textbook and complete exercises 1 to 3 of "Investigation 1: Measuring in the Classroom." **Note:** The textbook calls the questions *exercises*. You may work with a partner or alone.


Take the measurements carefully to help decrease measurement errors. It is also good practice to measure an object more than once before reporting the measurement.

Turn to pages 11 and 12 of the textbook and complete exercises 1 to 5 of "Investigation 3: Measuring Beyond the Classroom." **Note:** You may work with a partner or alone. If you are working at home, measure similar distances in your house (consider the room you are working in as your classroom).

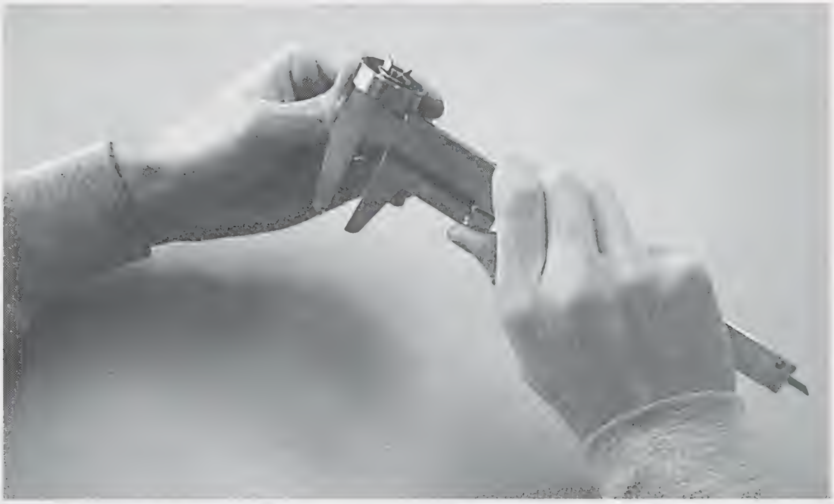
VERNIER CALIPERS

A vernier caliper is a tool for taking inside and outside measurements of items such as pipes, bolts, and machine parts.

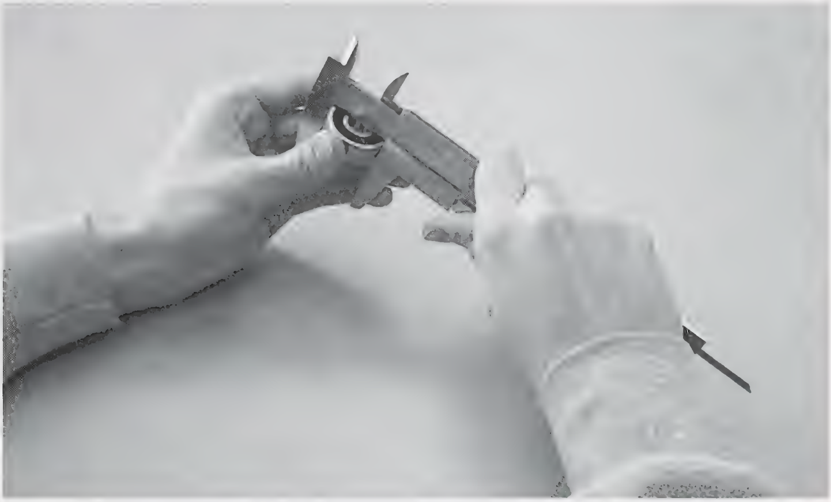
A vernier caliper has a scale, called a vernier scale, that slides along the main scale. The vernier scale indicates the subdivisions of the main scale. There is usually a scale along each edge of the main scale and the vernier scale. Often one edge has a metric scale and the other edge has an imperial scale.

Take out your vernier calipers and examine the device.

To find the inside measurements of a pipe or machine part, place the vernier caliper as shown in the following photograph.



To find the outside measurements of a pipe or machine part, place the vernier caliper as shown in the following photograph.



Turn to pages 389 and 390 of the textbook and work through Utility 2.

For some extra examples on using a vernier caliper, work with the vernier caliper segment (Java™ applet) on the companion CD.

6. a. Give the metric reading for the two vernier calipers shown on page 391 of the textbook. **Note:** Each division on the scale of the main arm is 0.1 cm. Each division on the vernier scale is 0.01 cm, or 0.1 mm.

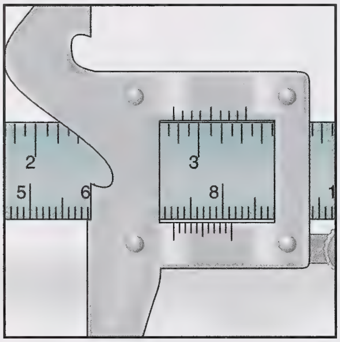
   b. Give the imperial reading for the two vernier calipers shown on page 391 of the textbook. **Note:** Each division on the scale of the main arm is 0.05 of an inch. Each division on the vernier scale is 0.01 of an inch.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 72–73.

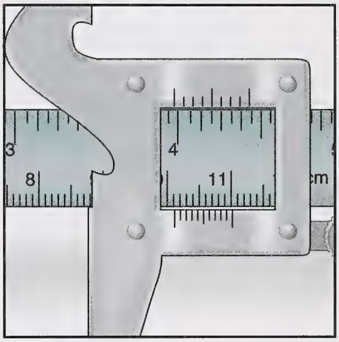
Vernier calipers have different scales. To discover how to read a vernier caliper in which the divisions of an inch on the main arm are common fractions, turn to page 392 of the textbook and work through the example.

7. Read the imperial measurement in each of the following vernier calipers. **Note:** Each division of the scale on the main arm is $\frac{1}{16}$ of an inch. Each division on the vernier scale is $\frac{1}{128}$ of an inch.

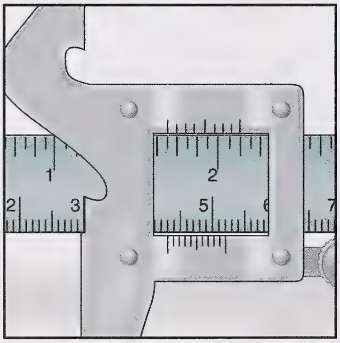
a.



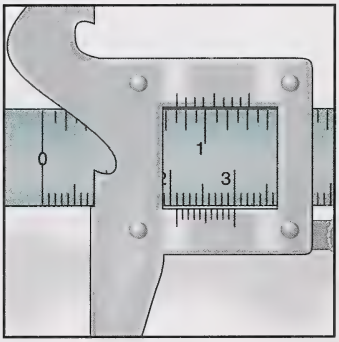
b.



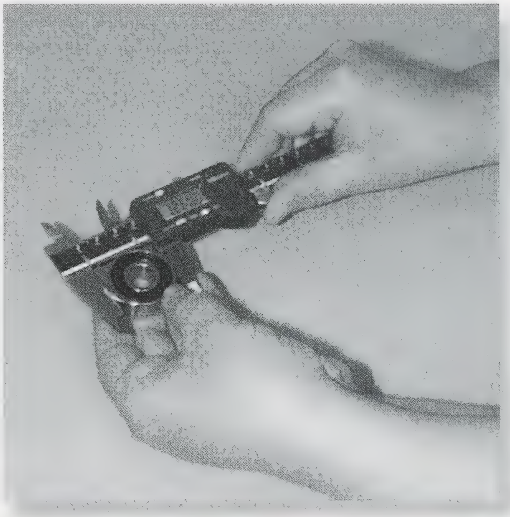
c.



d.

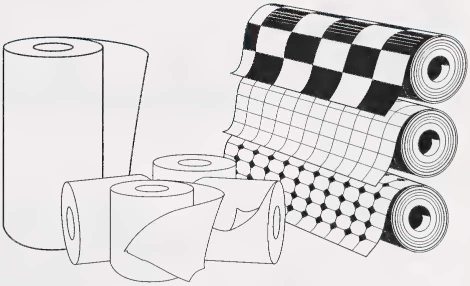


8. The following photograph shows a vernier caliper with a digital display. Why would it be helpful to have a digital display?



Compare your responses with the suggested answers in the Appendix, Activity 1, pages 73–74.

Now, you will apply what you have learned about vernier calipers. Obtain several different cardboard tubes. You may use the tubes from rolls of wrapping paper, toilet paper, paper towel, or aluminum foil. (The rolls do not have to be empty.)

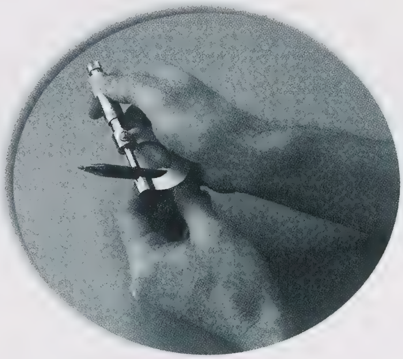


Measure the outside and inside diameters of each of the tubes with a vernier caliper. **Note:** Read the metric scale on the vernier caliper.

Be sure to adjust the jaws of the vernier caliper on the object until the jaws almost contact the sides of the object. The jaws should fit firmly but not tightly. Never force a vernier caliper on the object you are measuring.

MICROMETERS

A micrometer is a measuring tool used to measure the outside measurements of small objects, such as small machine parts.



Take out your micrometer and examine it. To discover how to read a micrometer, turn to page 393 of the textbook and work through Utility 3.

If you have access to the Internet, you may wish to view a video segment on using a micrometer. Visit the following website:

http://142.232.132.50/netshow/default.htm

For some extra examples on using a micrometer, work with the micrometer segment (Java™ applet) on the companion CD.

9. Turn to page 395 of the texbook and read the two micrometers shown.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 74–75.

If a micrometer does not read 0.00 when it is closed, it should be **zero-corrected** each time it is used. Turn to page 396 of the textbook and read “Zero Correction.”

10. State the correction that should be made to the reading of a micrometer if the micrometer has the following reading when closed:

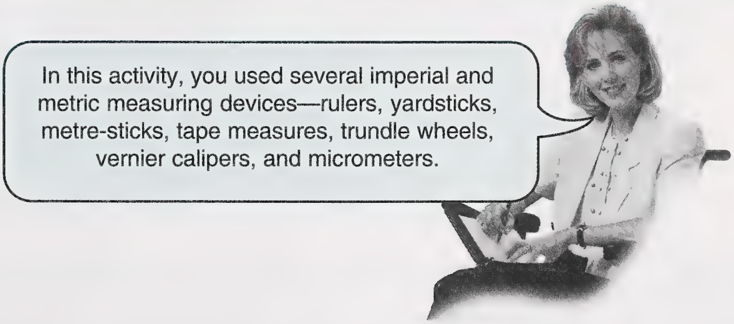
- **a.** 0.01 mm above zero
- **b.** 0.01 mm below zero

Compare your responses with the suggested answers in the Appendix, Activity 1, page 75.

Now, you will apply what you have learned about micrometers. Use a micrometer to measure the thickness of a pencil, a coin, an eraser, and a ruler.

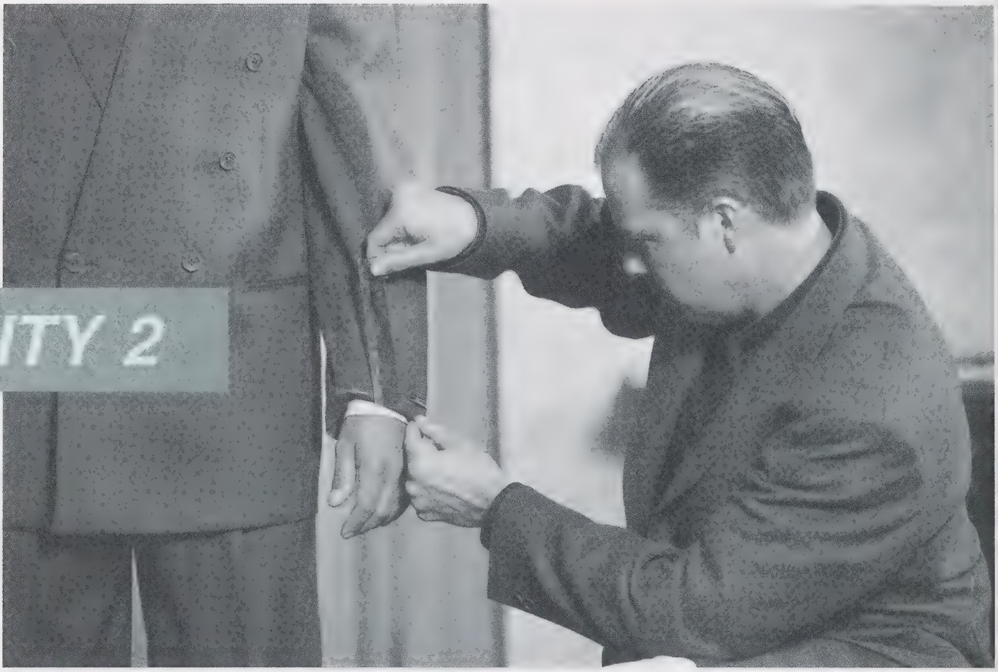
Remember to always close your micrometer with the thumbscrew.
SAFETY IS IN YOUR HANDS

LOOKING BACK


In this activity, you used several imperial and metric measuring devices—rulers, yardsticks, metre-sticks, tape measures, trundle wheels, vernier calipers, and micrometers.

11. Speak with several people in your community about the measuring devices they use in their daily lives. Then write a short report in the journal section of your mathematics binder.

Compare your response with the suggested answer in the Appendix, Activity 1, page 75.



ACTIVITY 2

Working with Measurements

Dale and his brother served as ushers at their sister's wedding last summer. They had to be fitted for new suits.

The Erfles want to fertilize their front lawn. They must measure the lawn and estimate the amount of fertilizer to buy.

The Jonks are getting new curtains for their kitchen. They must measure the windows and estimate the amount of material to buy.

Working with measurements is very important in the everyday world. In this activity, you will improve your skills in visualizing measurements, making estimates, and converting from one unit to another within the metric system or within the imperial system.

1. Turn to page 11 of the textbook and answer exercises 1 and 2 of "Investigation 2: Visualizing Measurement." **Note:** The textbook calls the questions *exercises*.

2. Turn to page 13 of the textbook and answer exercises 1 and 3 of "Discussing the Ideas." **Note:** If you are working at home, select a similar distance in your house for exercise 3.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 75–76.

The metre is the basic unit of length in the metric system; the yard is the basic unit of length in the imperial system.

To get a better sense of the size of a metre and a yard, take out a metre-stick and yardstick. Compare the sizes of a metre and a yard.

Units of Length

Metric	Imperial
1 kilometre (km) = 1000 metres	1 mile = 1760 yards
1 hectometre (hm) = 100 metres	1 furlong = 220 yards
1 decametre (dam) = 10 metres	1 rod = $5\frac{1}{2}$ yards
1 metre (m)	1 yard
1 decimetre (dm) = 0.1 metres	1 foot = $\frac{1}{3}$ of a yard
1 centimetre (cm) = 0.01 metres	1 inch = $\frac{1}{36}$ of a yard
1 millimetre (mm) = 0.001 metres	
1 micrometre (μm) = 0.0001 metres	

3. Copy and complete the following tables:

Yards	1		6		10	
Feet	3	6		10		25

Inches	12	36		18	64	
Feet	1		4			9

4. Turn to page 19 of the textbook and answer exercise 2 of “Discussing the Ideas.”

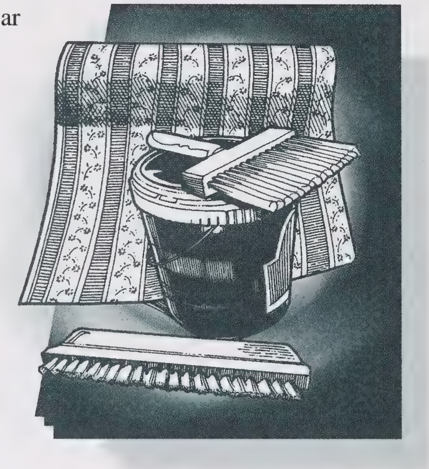
Compare your responses with the suggested answers in the Appendix, Activity 2, page 76.

Although it is conventional to measure lengths in feet and inches, it is often necessary in problem-solving situations to rewrite all the measurements in either inches or feet.

Work through the following example.

Example

Suzanne is redecorating a rectangular bedroom that is 12 feet 8 inches by 8 feet 4 inches. She wants to run a wallpaper border around the perimeter of the room, just below the ceiling. The wallpaper border comes in rolls that are 8 feet long. How many rolls of wallpaper will Suzanne need? **Note:** Ignore any wastage due to matching the pattern on the paper.



Solution

Step 1: Rewrite the measurements in feet. **Note:** In this problem, it is easier to use fractions. In other problems, it may be easier to use decimals.

$$12 \text{ feet } 8 \text{ inches} = 12\frac{8}{12} \text{ feet} \qquad 8 \text{ feet } 4 \text{ inches} = 8\frac{4}{12} \text{ feet}$$

$$= 12\frac{2}{3} \text{ feet} \qquad = 8\frac{1}{3} \text{ feet}$$

Step 2: Determine the perimeter, *P*, of the room. **Hint:** The perimeter is the sum of all the sides.

$$\begin{aligned} P &= \ell + \ell + w + w \\ &= 12\frac{2}{3} + 12\frac{2}{3} + 8\frac{1}{3} + 8\frac{1}{3} \\ &= 40 + \frac{6}{3} \\ &= 40 + 2 \\ &= 42 \end{aligned}$$

The perimeter is 42 feet.

You may also use vertical addition.

$$\begin{array}{r} 12\frac{2}{3} \\ 12\frac{2}{3} \\ 8\frac{1}{3} \\ 8\frac{1}{3} \\ \hline 40\frac{6}{3} \end{array} = 40 + 2$$

$$= 42$$

Step 3: Determine the number of rolls needed.

$$42 \div 8 = 5.25$$

Because Suzanne cannot buy part of a roll, she will need 6 rolls of wallpaper.

5. Andrea decides to paint the ceiling in her bedroom, which is 12 feet 6 inches by 10 feet 3 inches. The ceiling will require two coats of paint.

   If one can of paint covers 40 square feet, how many cans of paint will Andrea need?

6. Juan decides to panel the walls in his bedroom. He uses a tape measure and determines that his bedroom is 13 feet 6 inches by 11 feet 4 inches. If the panels are 6 feet wide, how many panels will Juan need?

   Note: The walls in Juan's bedroom are a standard height and the panels are the same height. Ignore the wastage because of any doors and windows.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 77–78.

When calculating the amount of wallpaper you need to buy, remember that you must match the pattern on the paper. This will result in some wastage.

If you have access to the Internet, you can get information on how to estimate the amount of wallpaper that is needed for a room by visiting the following website:

http://www.pdra.org/dr/articles/wallpaper.html

You can also visit a store that sells wallpaper. Stores selling wallpaper use a standard usable yield chart as a general guideline for determining the amount of wallpaper required for standard, eight-foot-high walls.

Length of Repeat Pattern (inches)	Usable Yield of Wallpaper (square feet)
0–6	25
7–12	22
13–18	20
19–23	18

To determine the number of rolls of wallpaper you require, follow these steps:

Step 1: Find the square footage of the walls of the room.

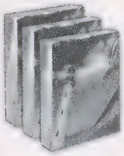
Step 2: Identify the length of the repeat pattern on the wallpaper. (This is generally indicated on the sample of paper and on each roll.) Then use the chart to find the usable yield of each roll.

Step 3: Divide the area of your room by the usable yield. Round **up** to the nearest whole number. The answer gives you the number of required **single** rolls. **Note:** Wallpaper is often sold in double rolls; if this is the case with the paper you choose, you must divide the answer by 2. Again, round **up** to the nearest whole number because partial rolls of paper are not sold.

7. William wants to wallpaper his bedroom. He selects a wallpaper with a 5-inch repeat pattern, which is sold in double rolls. If the room has a total of 420 square feet, how many rolls of wallpaper will William need?

Compare your response with the suggested answer in the Appendix, Activity 2, page 78.

WORKING WITH AREA



Turn to page 12 of the textbook and read the beginning of "Investigation 4: Working with Area." This investigation will help you visualize various area measurements.

Gather the materials needed for the investigation and complete exercise 1 of "Investigation 4: Working with Area."

8. Turn to page 12 of the textbook and answer exercise 2 of "Investigation 4: Working with Area."

Complete exercise 3 of "Investigation 4: Working with Area."

9. Turn to page 12 of the textbook and answer exercise 4 of "Investigation 4: Working with Area."

Complete exercises 5 and 6 of "Investigation 4: Working with Area."

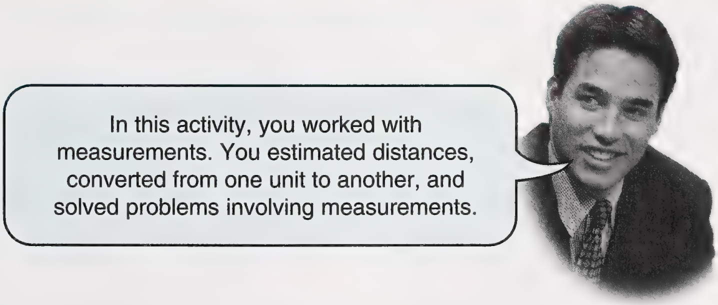
Compare your responses with the suggested answers in the Appendix, Activity 2, page 79.

Turn to page 14 of the textbook and read the opening problem of "Measurement Conversions." Then turn to page 17 of the textbook and work through Example 1.

10. Turn to page 20 of the textbook and answer exercise 4.a. of "Exercises: Checking Your Skills." Give your answer to the nearest square yard.

Compare your response with the suggested answer in the Appendix, Activity 2, page 79.

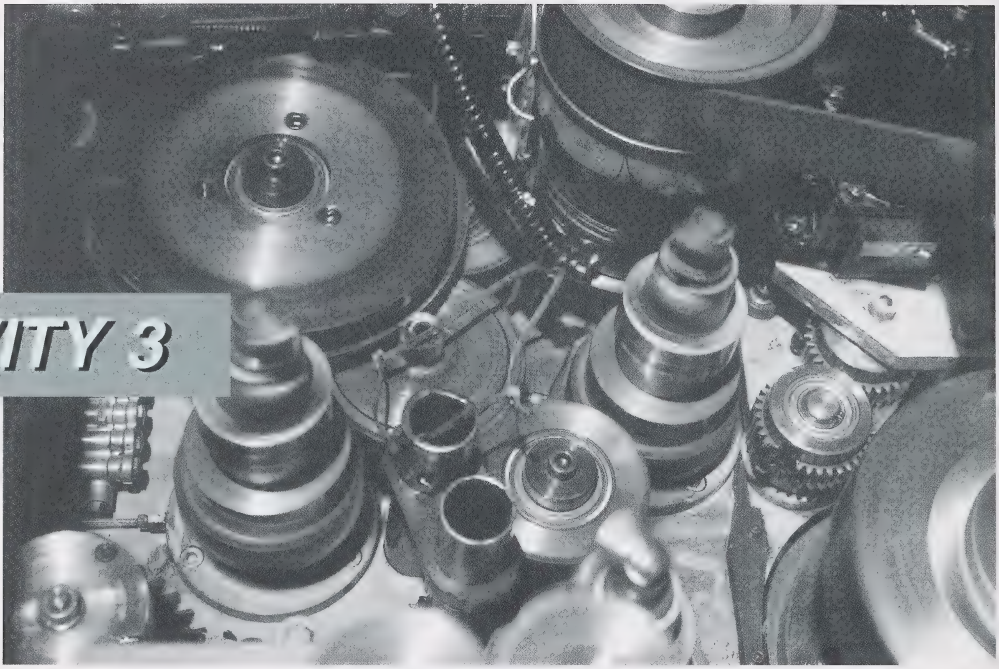
LOOKING BACK


In this activity, you worked with measurements. You estimated distances, converted from one unit to another, and solved problems involving measurements.

11. Turn to page 13 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 79.

ACTIVITY 3



Precision and Accuracy

The photograph shows a complex machine with many movable parts. If one of the machine parts breaks, the task of finding a replacement part that fits exactly is not easy. A small discrepancy in size will result in a poor fit, and the machine will not operate properly.

Your measurement tasks in this module will not be as demanding. However, you should strive for **accuracy** and **precision** in your measurements.

The accuracy of a measurement refers to how close the measurement value is to the true or accepted value. For example, if you use a balance to find the mass of a known standard 100-g mass and you get a reading of 78 g, your measurement is not accurate.

Precision refers to how close together a group of measurements actually are to each other. For example, if you measure your little sister's height on a growth chart, and you get readings of 3' 4", 3' 8", and 3' 6", your measurements were not very precise.

One important distinction between accuracy and precision is that accuracy can be found by only one measurement; precision can only be found by multiple measurements.

An analogy using darts may help you understand the difference between accuracy and precision. Imagine a person throwing several darts trying to hit the bull's-eye. The closer each dart is to the bull's-eye, the more accurate the throw. If all the darts land very close together, the dart player has thrown the darts with precision.

1. Turn to page 27 of the textbook and answer exercise 3 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, page 80.

Turn to page 22 of the textbook and read the opening problem. Respond to the two questions posed before reading the remainder of the page.

Then turn to page 23 of the textbook and read from the top of the page to Example 1. This gives a second meaning for the word *precision* and explains the concept of **uncertainty**.

Remember that every measuring device has its limitations, and every measurement is an approximation. By using a more-precise measuring device, you can get closer to the true value of a quantity, but there will always be some uncertainty.

Consider the following examples.

Example

Matthew measures the thickness of an eraser with a ruler that has a precision of 1 mm. Matthew reports that the eraser is 6 mm thick. Is this the true thickness of the eraser?

Solution

No, the thickness of the eraser is 6 mm, plus or minus 0.5 mm. In other words, the thickness of the eraser is between 5.5 mm and 6.5 mm.

Example

Rachel measures the thickness of an eraser with a micrometer that has a precision of 0.01 mm. Rachel reports that the eraser is 5.64 mm thick. Is this the true thickness of the eraser?

Solution

No, the thickness of the eraser is 5.64 mm, plus or minus 0.005 mm. In other words, the thickness of the eraser is between 5.635 mm and 5.645 mm.

Turn to page 23 of the textbook and work through parts a., b., and c. of "Example 1: Determining precision and accuracy of a measurement."

2. Rosemary measures the width of a book with a ruler whose smallest division is 0.1 of an inch. She reports the width as 8.6 inches.

   a. What is the precision of the ruler?
   b. What is the uncertainty of the measurement?
   c. Between what two measurements does the true width of the book lie?

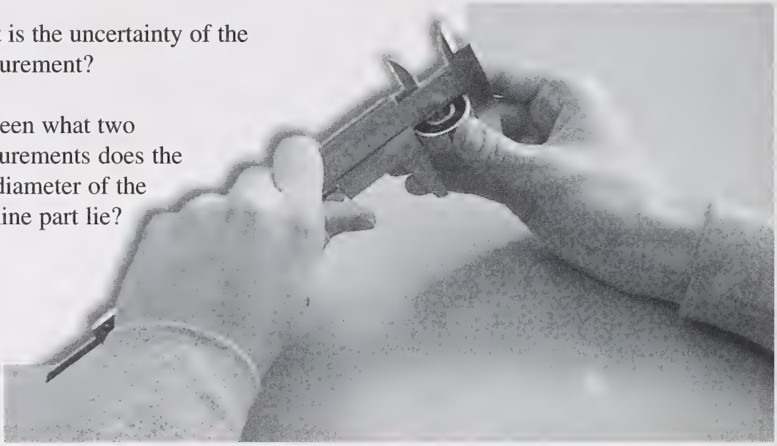
3. Frank measures his driveway with a tape measure whose smallest division is $\frac{1}{8}$ of an inch. He reports the length of the driveway as 22 feet $\frac{5}{8}$ inch.

   a. What is the precision of the tape measure?
   b. What is the uncertainty of the measurement?
   c. Between what two measurements does the true length of the driveway lie?

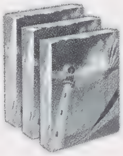
4. Maureen measures the outside diameter of a machine part with a vernier caliper that has a precision of 0.001 of an inch. She reports the outside diameter as 6.124 inches.

   a. What is the uncertainty of the measurement?

   b. Between what two measurements does the true diameter of the machine part lie?



Compare your responses with the suggested answers in the Appendix, Activity 3, page 80.



Turn to page 23 of the textbook and work through part d. of "Example 1: Determining precision and accuracy of a measurement."

5. Does the answer to part d. of Example 1 seem reasonable? Explain.

Compare your response with the suggested answer in the Appendix, Activity 3, page 80.

Manufacturers usually work from a plan that specifies the dimensions of the parts of the product they are producing. Because the engineers or technicians who develop the plans recognize that in mass production there will be some variances in the size of the parts, they often specify the **tolerances**.

6. a. The plans for a particular pipe state that the pipe should have a diameter of 4 mm with a tolerance of ±0.1 mm. State the minimum acceptable diameter of the pipe.

b. The plans for a box state that the box should be 11 inches long, $4\frac{1}{4}$ inches wide, and $5\frac{3}{4}$ inches high. Each dimension can be $\frac{1}{8}$ of an inch greater than the specified dimension. State the maximum acceptable length, width, and height of the box.

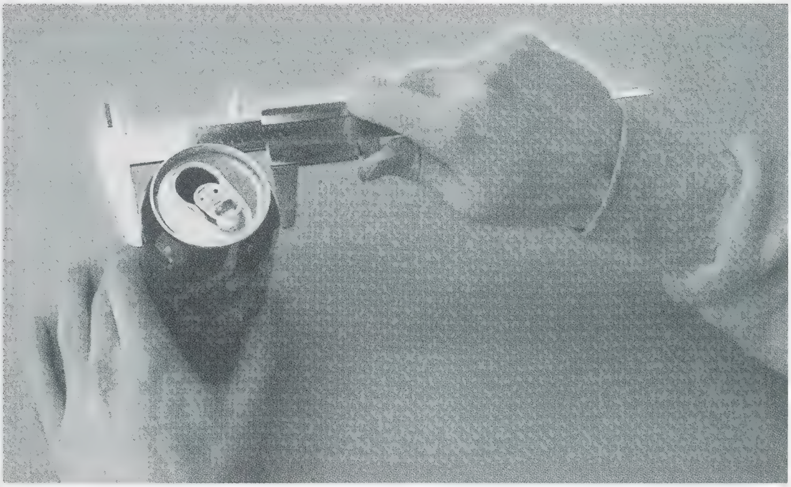
7. An electrical device is designed to run on 120 volts, ±10%. Between what voltages is the electrical device designed to run?

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 80–81.

You can now apply what you have learned about tolerances.

Obtain a pop can and a vernier caliper. Then complete the following steps:

Step 1: Measure the diameter and height of the can. **Note:** Use the metric scale on the vernier caliper. Give the measurements in centimetres.



Step 2: Draw a sketch of a rectangular box that will hold the can. On the sketch, include the dimensions of the box.

Step 3: Use the sketch to construct a net for the box. **Note:** The box should have a top, bottom, and four sides.

Note: If you need to review what a net is, turn to page 425 of the textbook and use the Student Reference.

Step 4: Cut out the net and assemble the box. Does the can fit snuggly in the box?

Small measurement errors are acceptable. For example, the length and width of the box may not be less than the diameter of the can, but may be 0.2 cm greater than the diameter of the can. Similarly, the height of the box may not be less than the height of the can, but may be 0.2 cm greater than the height of the can.

LOOKING BACK

In this activity, you used the rules for precision and accuracy to express final answers. You also considered measurement error and tolerances.

8. Turn to page 27 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 3, page 81.

ACTIVITY 4

Volume and Surface Area of a Sphere

There are many spherical, or ball-like, objects in the world. Baseballs, volleyballs, and basketballs are spheres. Earth, the Moon, and the Sun are spherical. What other spheres can you name?

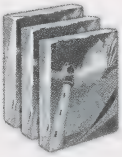
VOLUME OF A SPHERE

You will now complete an investigation on the volume of a sphere. Turn to page 30 of the textbook and read the beginning of "Investigation: The Volume of a Sphere." Then gather the materials that you need for the investigation.

Complete exercises 1 and 2 of "Investigation: The Volume of a Sphere."

1. Turn to page 33 of the textbook and answer exercise 1 of "Discussing the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 81.

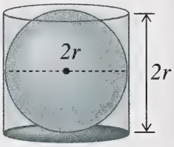


Turn to page 31 of the textbook and complete exercise 3 of "Investigation: The Volume of a Sphere."

2. Turn to page 31 of the textbook and answer exercises 4, 5, and 6 of "Investigation: The Volume of a Sphere."

Compare your responses with the suggested answers in the Appendix, Activity 4, page 81.

In the investigation, you discovered that the volume of a sphere is $\frac{2}{3}$ the volume of a cylinder. You can use this relationship to derive the formula for the volume of a sphere.



Suppose that the can (cylinder) and the ball (sphere) each have a radius *r*. Then the diameter of each is 2*r*.

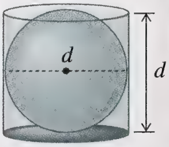
The volume of a cylinder is equal to the area of the base (*B*) times the height (*H*) of the cylinder. The volume of a cylinder can be expressed by the following formula:

$$\begin{aligned} V &= BH \\ &= \pi r^2 (2r) \\ &= 2\pi r^3 \end{aligned}$$

Because the volume of the ball (sphere) is $\frac{2}{3}$ the volume of the can (cylinder), the volume of a sphere can be expressed by the following formula:

$$\begin{aligned} V &= \frac{2}{3}(2\pi r^3) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

The volume of a sphere with a radius of *r* is $V = \frac{4}{3}\pi r^3$.



Now, suppose the can and the ball each have a diameter *d*. Then the height of the can is *d*.

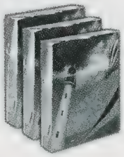
The volume of a cylinder is equal to the area of the base (*B*) times the height (*H*) of the cylinder. The volume of a cylinder can be expressed by the following formula:

$$\begin{aligned} V &= BH \\ &= \pi\left(\frac{d}{2}\right)^2 d \\ &= \pi\left(\frac{d^2}{4}\right) d \\ &= \frac{\pi d^3}{4} \end{aligned}$$

Because the volume of the ball (sphere) is $\frac{2}{3}$ the volume of the can (cylinder), the volume of a sphere can be expressed by the following formula:

$$
\begin{aligned}
V &= \frac{\overset{1}{\cancel{2}}}{3}\left(\frac{\pi d^3}{\underset{2}{\cancel{4}}}\right) \\
&= \frac{1}{6}\pi d^3
\end{aligned}
$$

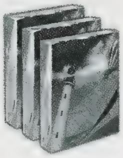
The volume of a sphere with a diameter of d is $V = \frac{1}{6}\pi d^3$.



3. Turn to page 33 of the textbook and answer exercise 2 of "Discussing the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 81.

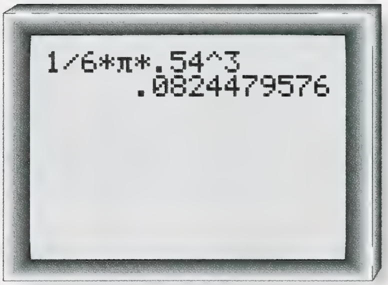
When you calculate the volume of a sphere, you must remember the rules for the order of operations.



Turn to pages 32 of the textbook and work through Example 1.

To use a graphing calculator for the calculations in Example 1, press the following key sequence:

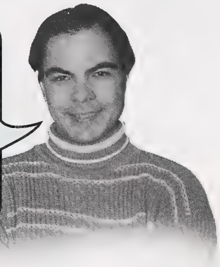
1 ÷ 6 × 2nd [π] × • 5 4 ^ 3 ENTER



```
1/6*π*.54^3
           .0824479576
```

Notice that the volume of steel in Example 1 has been rounded to 0.082 cm^3. It would have been unreasonable to express the volume to 10 decimal places when the diameter was only measured to 2 decimal places.

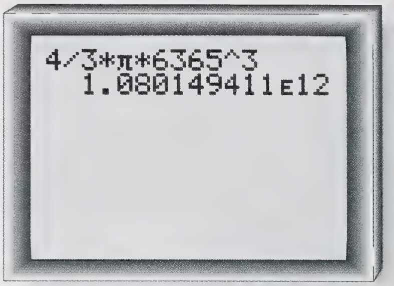
Rules for rounding in Applied Mathematics 10 are fairly flexible. If you know the rules for significant digits from science, you should use these rules. However, if you do not know these rules, simply round your answer to a reasonable place value.



Turn to page 33 of the textbook and work through Example 2.

To use a graphing calculator for the calculations in Example 2, press the following key sequence:

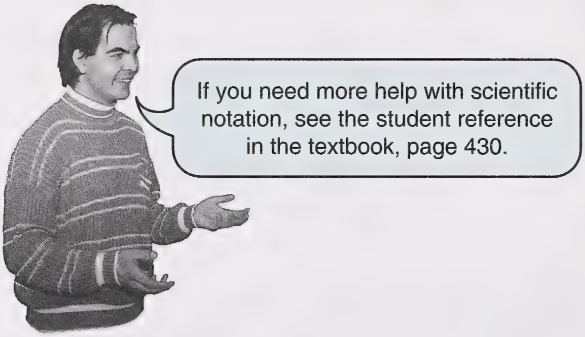
[4] [÷] [3] [×] [2nd] [π] [×]
[6] [3] [6] [5] [^] [3] [ENTER]



```
4/3*π*6365^3
  1.080149411E12
```

Because the volume in Example 2 is a very large quantity, the answer is given in scientific notation. The expression "1.080 149 411 E 12" means $1.080\ 149\ 411 \times 10^{12}$.

Notice that the volume has been rounded to 1.1×10^{12} km^2. The standard form of 1.1×10^{12} km^2 is 1 100 000 000 000 km^2.


If you need more help with scientific notation, see the student reference in the textbook, page 430.

4. Turn to page 34 of the textbook and answer exercises 1 to 4 of "Exercises: Checking Your Skills."

5. Turn to page 35 of the textbook and answer "Communicating the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 82–85.

SURFACE AREA OF A SPHERE

You will now complete an investigation on the surface area of a sphere. Turn to page 36 of the textbook and read the beginning of "Investigation: Surface Area of a Sphere."

Gather the materials that you need for the investigation. Then turn to page 37 of the textbook and complete exercises 1 and 2 of "Investigation: Surface Area of a Sphere."

6. Turn to page 38 of the textbook and answer exercise 1 of "Discussing the Ideas."

Turn to page 37 of the textbook and complete exercise 3 of "Investigation: Surface Area of a Sphere."

7. Turn to page 38 of the textbook and answer exercises 2 and 3 of "Discussing the Ideas."

Turn to page 37 of the textbook and complete exercise 4 of "Investigation: Surface Area of a Sphere."

8. Turn to page 37 of the textbook. Answer exercises 6, 7, and 8 of "Investigation: Surface Area of a Sphere."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 85–86.

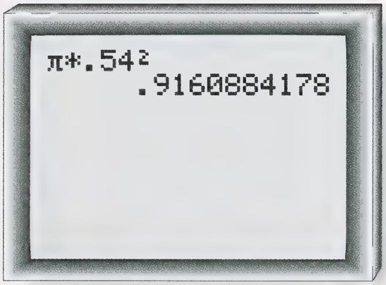
In the preceding investigation, you examined the relationship between the surface area of an orange and the diameter of an orange. This relationship is the same for all spheres:

- The surface area of a sphere with a radius of r is $A = 4\pi r^2$.
- The surface area of a sphere with a diameter of d is $A = \pi d^2$.

Turn to page 38 of the textbook and work through "Example: Determine the surface area of a sphere, given its diameter."

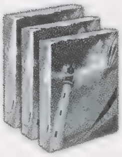
To use a graphing calculator for the calculations in this example, press the following key sequence:

2nd [π] × • 5 4 x^2
ENTER



```
π*.54²
          .9160884178
```

Note: Alternatively, you can press ^ 2 instead of the x^2 key.



9. Turn to page 39 of the textbook and answer exercises 1 and 2 of "Exercises: Checking Your Skills."

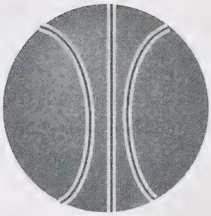
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 86–87.

The following example is a multipart problem. The general rule for multipart problems is **not** to use the rounded answer to the first part of the problem in the calculations in the second part of the problem. Instead of inputting the rounded answer, keep the calculator running and press 2nd [ANS].

Work through the example.

Example

A basketball has a circumference of 76 cm.

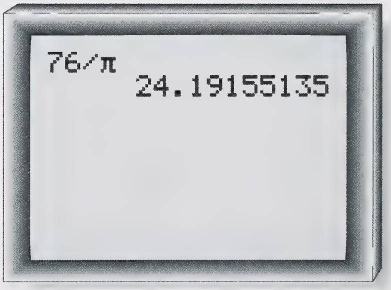


a. What is its diameter?
b. What is its surface area?

Solution

a. Find the diameter.

$$C = \pi d$$
$$76 = \pi d$$
$$\frac{76}{\pi} = d$$
$$d \doteq 24$$



```
76/π
          24.19155135
```

The diameter is approximately 24 cm.

Note: This is a rounded answer. Do not use the rounded answer in part b. Instead, keep the calculator running and use the answer key in part b.

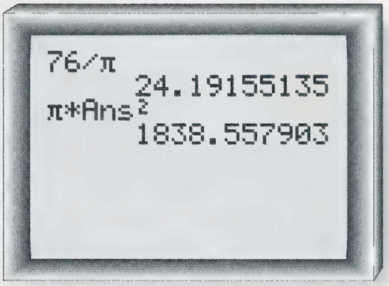
b. Find the surface area.

$$A = \pi d^2$$
$$\doteq 1800$$

The surface area is approximately 1800 cm^2.

2nd [π] × 2nd [ANS]

x^2 ENTER



```
76/π
             24.19155135
π*Ans²
             1838.557903
```

10. Turn to page 39 of the textbook and answer exercises 3, 4, and 5 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 88–90.

LOOKING BACK

In this activity, you explored the volume of a sphere and the surface area of a sphere.

11. Turn to page 40 of the textbook and answer "Communicating the Ideas."

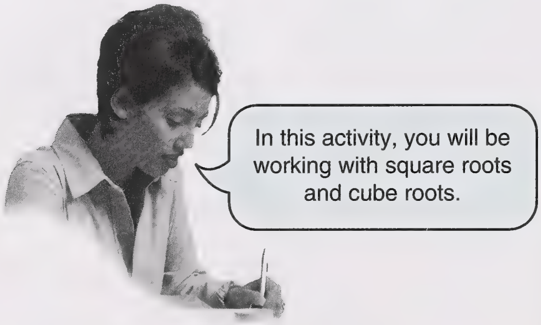
Compare your response with the suggested answer in the Appendix, Activity 4, page 91.

ACTIVITY 5

Square Roots and Cube Roots

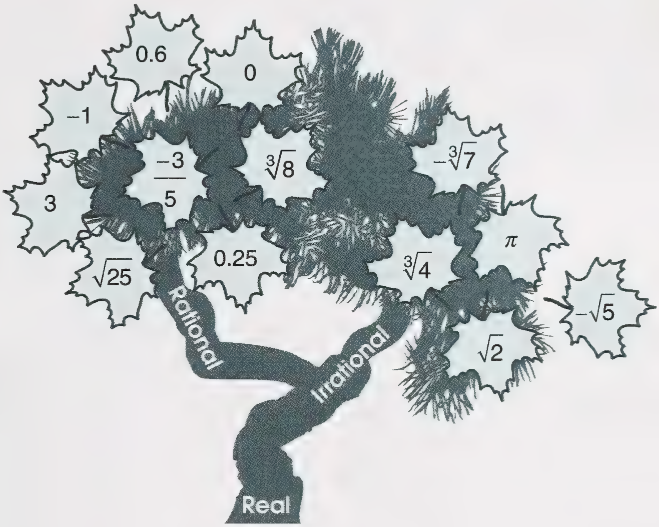
There are many opposite actions in life. Examples include tying and untying your shoes, opening and closing a door, and folding and unfolding a towel.

In mathematics, there are also many opposite actions. For example, subtracting is the opposite of adding, dividing is the oppostite of multiplying, taking the square root is the opposite of squaring, and taking the cube root is the opposite of cubing.


In this activity, you will be working with square roots and cube roots.

SQUARE ROOTS

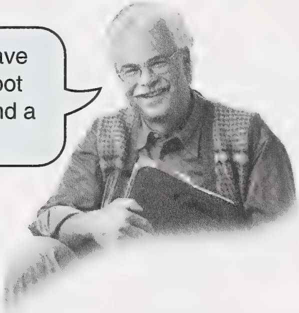
The following diagram depicts the real number system as a tree. The tree has two main branches: the set of rational numbers and the set of irrational numbers. Examples of numbers in these sets are shown as leaves on the tree.



The square root of a perfect square is a rational number. The square root of a non-perfect square is an irrational number.

Remember that all positive numbers have two square roots—a positive square root (also called the principal square root) and a negative square root.

The square root symbol, $\sqrt{}$, is used to denote the positive square root.



Work through the following example. It illustrates how to express the positive square root of non-perfect squares as a decimal approximation.

Example

Estimate the value of each of the following square roots. Then use a calculator to find the decimal approximation. Round each calculated value to the nearest hundredth.

a. $\sqrt{1824}$ **b.** $\sqrt{792}$ **c.** $\sqrt{0.045}$ **d.** $\sqrt{0.000\ 41}$

Solution

a. Estimated Value

$$\sqrt{824} = \sqrt{\underset{\underset{4}{\downarrow}}{18}\underset{\underset{0}{\downarrow}}{24}}$$

$$\doteq 40$$

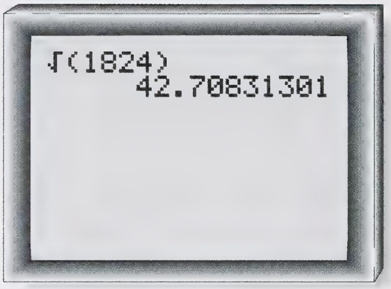
Calculated Value

Using your calculator, press the following:

2nd [√‾] ← This key sequence pastes √‾(onto the screen.

1 8 2 4) ENTER

Remember to insert the closing parenthesis.



```
√(1824)
         42.70831301
```

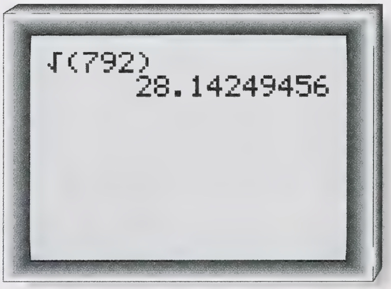
$\therefore\ \sqrt{1824} \doteq 42.71$

b. Estimated Value

$$\sqrt{792} = \sqrt{\underset{\underset{2}{\downarrow}}{07}\underset{\underset{0}{\downarrow}}{92}}$$

$$\doteq 20$$

Calculated Value



```
√(792)
         28.14249456
```

$\therefore\ \sqrt{792} \doteq 28.14$

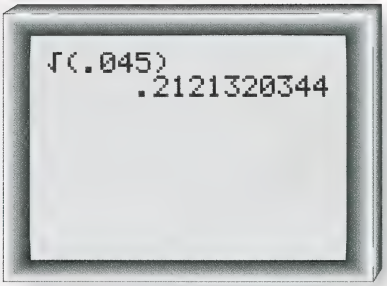
c. Estimated Value **Calculated Value**

$$\sqrt{0.045} = \sqrt{0.0450}$$

↓ ↓

.2 0

$\doteq 0.20$



```
√(.045)
          .2121320344
```

$\therefore \sqrt{0.045} \doteq 0.21$

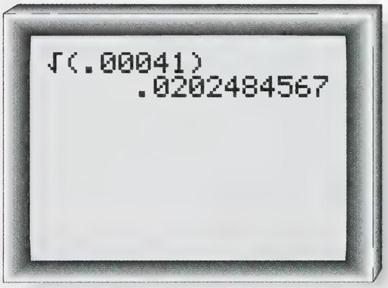
d. Estimated Value **Calculated Value**

$$\sqrt{0.000\ 41} = \sqrt{0.000\ 410}$$

↓ ↓ ↓

.0 2 0

$\doteq 0.020$



```
√(.00041)
          .0202484567
```

$\therefore \sqrt{0.000\ 41} \doteq 0.02$

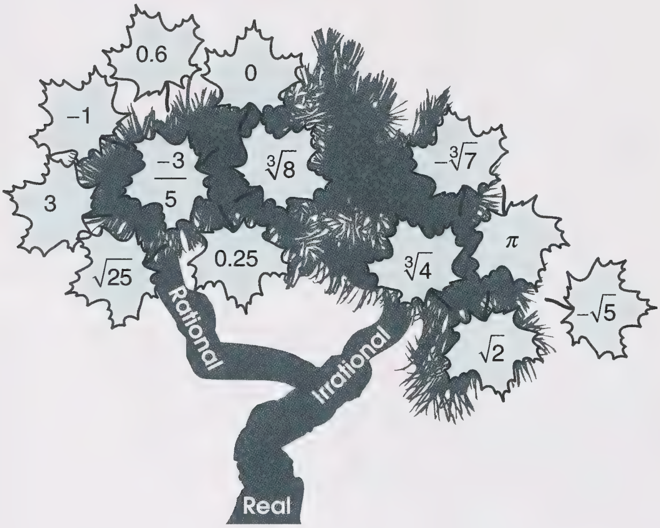
1. Estimate the positive square root of each of the following numbers. Then use your graphing calculator to determine the positive square root; round each answer to the nearest hundredth.

a. 9821
b. 153
c. 42 563
d. 0.045
e. 0.74
f. 0.0142

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 91–93.

CUBE ROOTS

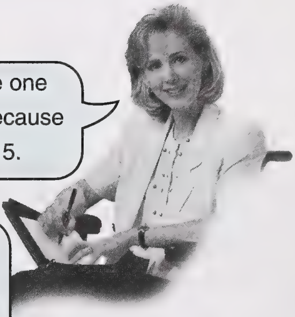
Recall the diagram depicting the real number system as a tree. The tree has two main branches: the set of rational numbers and the set of irrational numbers. Examples of numbers in these sets are shown as leaves on the tree.



The cube root of a perfect cube is a rational number. The cube root of a non-perfect cube is an irrational number.

Remember that all positive numbers have one cube root—a positive root. For example, because $5 \times 5 \times 5 = 125$, the cube root of 125 is 5.

Likewise, all negative numbers have one cube root—a negative root. For example, because $(-5)\times(-5)\times(-5) = -125$, the cube root of -125 is -5.



Work through the following example. It illustrates how to express the cube root of a non-perfect cube as a decimal approximation.

Example

Estimate the value of $\sqrt[3]{70}$. Then use a calculator to find the decimal approximation, rounding to the nearest tenth.

Solution

Estimated Value

Because $4 \times 4 \times 4 = 64$ and $5 \times 5 \times 5 = 125$, $\sqrt[3]{70}$ is between 4 and 5.

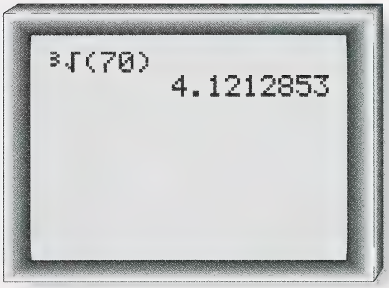
Calculated Value

Using your graphing calculator, press the following:

MATH 4 ← This key sequence pastes $\sqrt[3]{}($ onto the screen.

7 0) ENTER

Remember to insert the closing parenthesis.



```
³√(70)
              4.1212853
```

$\therefore \sqrt[3]{70} \doteq 4.1$

2. Estimate the cube root of each of the following numbers. Then use your graphing calculator to determine the cube root. Round each answer to the nearest hundredth.

a. 7

b. 89

c. 942

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 93–94.

SOLVING PROBLEMS

You will now solve problems involving formulas in which one of the variables is squared or cubed.

Example

If the surface area of a sphere is 100 cm^2, estimate and then calculate the diameter of the sphere. Round the calculated answer to the nearest tenth of a centimetre.

Solution

Estimated Value

$$A = \pi d^2$$

$$100 \doteq 3d^2 \quad \longleftarrow \text{ π is about 3.}$$

$$\frac{100}{3} \doteq d^2$$

$$33 \doteq d^2$$

$$d \doteq 5.5 \quad \longleftarrow \text{ The square root of 33 is between 5 and 6.}$$

Calculated Value

$$A = \pi d^2$$

$$100 = \pi d^2$$

$$\frac{100}{\pi} = d^2$$

$$d = \sqrt{\frac{100}{\pi}}$$

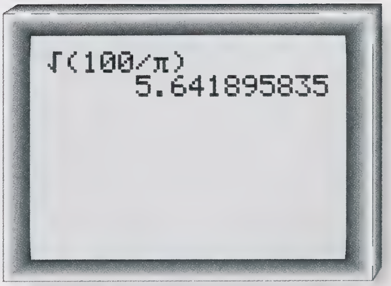
$$d \doteq 5.6$$

2nd [√] ← This key sequence pastes √(on the screen.

1 0 0 ÷

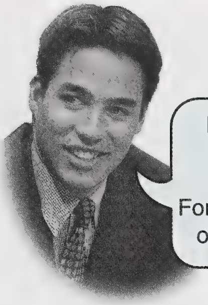
2nd [π]) ENTER

↑ Remember to insert the closing parenthesis.



```
√(100/π)
            5.641895835
```

The diameter of the sphere is approximately 5.6 cm.



For **estimations** involving π, you may use 3 as an approximate value of π.

For **calculations** involving π, you should use the π key on your calculator instead of the rounded value of π.

Example

If the volume of a sphere is 2 cubic inches, estimate and then calculate the diameter of the sphere. Round your answer to the nearest half inch.

Solution

Estimated Value

$$V = \frac{1}{6}\pi d^3$$

$$2 \doteq \frac{1}{6}(3)d^3$$

← π is about 3.

$$2 \doteq \frac{1}{2}d^3$$

$$4 \doteq d^3$$

← The cube root of 4 is between 1 and 2.

$$d \doteq 1.5$$

Calculated Value

$$V = \frac{1}{6}\pi d^3$$

$$2 = \frac{1}{6}\pi d^3$$

$$2 \times \frac{6}{\pi} = \frac{1}{6}\pi d^3 \times \frac{6}{\pi}$$

$$\frac{12}{\pi} = d^3$$

$$\sqrt[3]{\frac{12}{\pi}} = d$$

$$d \doteq 1.6$$

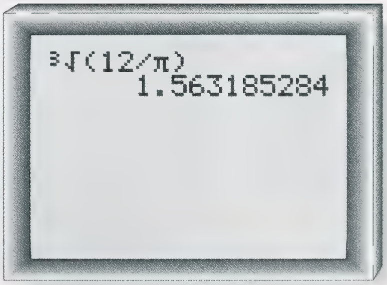
The diameter of the sphere is approximately 1.6 inches.

MATH 4 ← This key sequence pastes $\sqrt[3]{}($ onto the screen.

1 2 ÷

2nd [π]) ENTER

↑ Remember to insert the closing parenthesis.

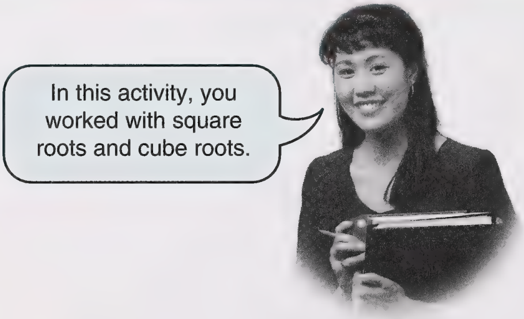


```
³√(12/π)
          1.563185284
```

3. Turn to page 44 of the textbook and answer exercises 2, 4, and 7 of "Exercises: Checking Your Skills." **Note:** Round each answer to the nearest tenth.

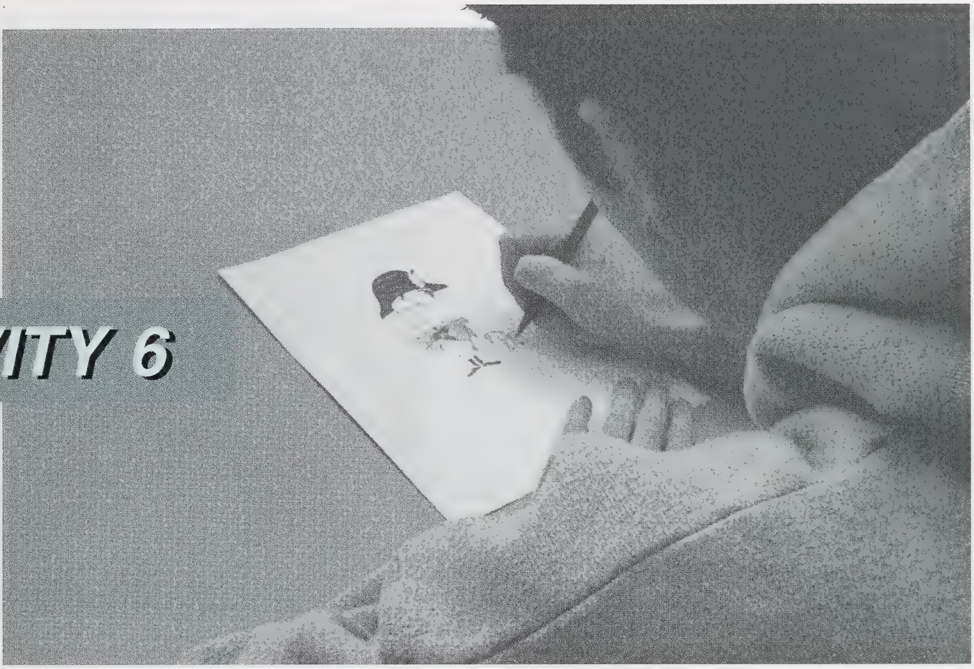
Compare your responses with the suggested answers in the Appendix, Activity 5, pages 94–95.

LOOKING BACK


In this activity, you worked with square roots and cube roots.

4. Turn to page 45 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 96.



ACTIVITY 6

Ratio and Proportion

Have you ever noticed that cartoonists often draw cartoon characters with heads that are not in proportion to the size of their bodies?

Unlike cartoonists, people who make scale models are careful to use ratio and proportion so that the models are **similar** to the real ones.

A model may be full-scale, a reduction, or an enlargement. Following are some examples:

- The Royal London Wax Museum in Victoria, British Columbia, has many full-scale (that is, 1:1) wax models of many famous people.

- The Alberta Provincial Museum had many scale models of insects at an exhibit called Bug World. The models were made to different scales. The model of the praying mantis in the photograph was made using an enlargement scale (60:1).

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- Over the years, Gerry and Bob Bell have made many scale models for the Old Strathcona Model and Toy Museum. The model of the RCMP officers and horses was made to a reduction scale (1:12).

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To review concepts related to scale, turn to page 46 of the textbook and read from the top of the page to Investigation 1.

Now, work through the following example.

Example

An architect uses a reduction scale of 1:36 to make a scale model of a house. If the actual house is 30 feet long, how long should the model house be?

Solution

Method 1: Completing the Proportion

Step 1: Convert feet to inches.

$$\begin{aligned} &1 \text{ foot} = 12 \text{ inches} \\ &\therefore\ 30 \text{ feet} = 360 \text{ inches} \end{aligned}$$

Step 2: Write a proportion.

$$\frac{\text{Drawn}}{\text{Actual}} \qquad \frac{1}{36} = \frac{x}{360}$$

Step 3: Use the proportion to calculate the length of the house in the drawing.

$$\frac{\text{Drawn}}{\text{Actual}} \qquad \frac{1}{36} = \frac{x}{360}$$

$$\frac{1}{36} \times 360 = \frac{x}{360} \times 360$$

← Multiply each side by 360.

$$10 = x$$

The model house should be 10 inches long.

Method 2: Multiplying the Actual Length by the Scale Factor

Step 1: Convert feet to inches.

$$\begin{aligned} &1 \text{ foot} = 12 \text{ inches} \\ &\therefore\ 30 \text{ feet} = 360 \text{ inches} \end{aligned}$$

Step 2: Determine the scale factor.

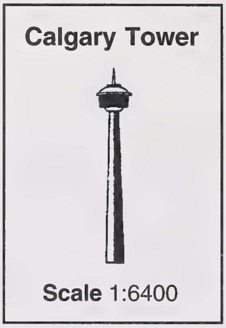
$$1:36 = \frac{1}{36}$$

Step 3: Multiply the actual length by the scale factor.

$$\frac{1}{36} \times 360 = 10$$

The model house should be 10 inches long.

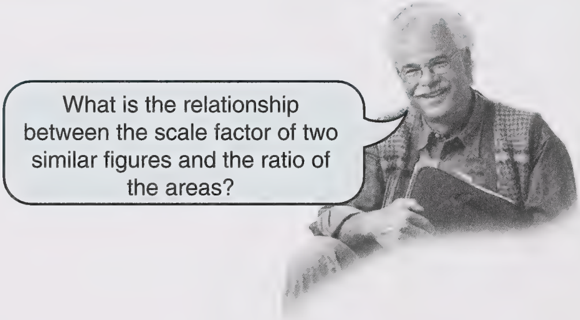
1. Following is a scale drawing of the Calgary Tower. What is the actual height (in metres) of the Calgary Tower?



2. Turn to page 50 of the textbook and answer exercise 1 of “Exercises: Checking Your Skills.” **Note:** Express the scale factors as either whole numbers or fractions in simplest form.

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 96–97.

THE RELATIONSHIP BETWEEN SCALE FACTOR AND AREA


What is the relationship between the scale factor of two similar figures and the ratio of the areas?

3. Turn to pages 46 and 47 of the textbook and answer exercises 1, 2, and 3 of “Investigation 1: The Relationship Between Scale Factor and Area.” **Note:** Express the scale factors as either whole numbers or fractions in simplest form.

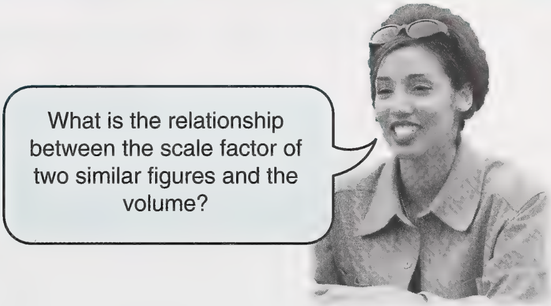
Compare your responses with the suggested answers in the Appendix, Activity 6, pages 97–98.

Turn to page 49 of the textbook and work through "Example 1: Use the relationship among linear scale factor, perimeter, and area."

4. Turn to page 52 of the textbook and answer exercise 6 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 98–99.

THE RELATIONSHIP BETWEEN SCALE FACTOR AND VOLUME


What is the relationship between the scale factor of two similar figures and the volume?

5. Turn to page 47 of the textbook and answer exercises 1 to 4 of "Investigation 2: The Relationship Between Scale Factor, Surface Area, and Volume." **Note:** Round all answers to the nearest hundredth.

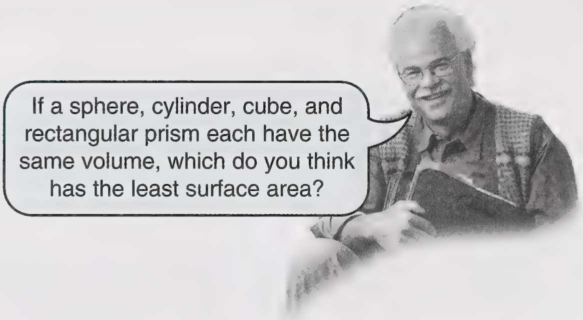
Compare your responses with the suggested answers in the Appendix, Activity 6, pages 99–101.

Turn to pages 49 and 50 of the textbook and work through "Example 2: Use the relationships among the linear scale factor, surface area, and volume of similar solids."

6. Turn to page 50 of the textbook and answer exercise 2 of "Exercises: Checking Your Skills."

Compare your response with the suggested answer in the Appendix, Activity 6, page 101.

THE RELATIONSHIP BETWEEN SURFACE AREA AND VOLUME


If a sphere, cylinder, cube, and rectangular prism each have the same volume, which do you think has the least surface area?

7. To answer the preceding question, turn to page 57 of the textbook. Answer exercise 1 of Part A of “What Should I Be Able To Do?” **Note:** If you need help with any formulas, check the Student Reference pages in the textbook. Keep your calculator running between parts. Give answers rounded to the nearest tenth.

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 102–104.

Why do you think bubbles are spherical? Take out the companion CD that accompanies this course and view the video clip on bubbles.

To find out more about why bubbles are round, visit the following Internet site:

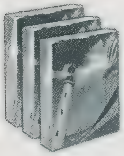
http://bubbles.org/html/round.htm

LOOKING BACK

In this activity, you examined ratio, proportion, and scale.

You discovered why sled dogs sleep curled up in a ball and why bubbles are spherical.

CORBIS/PAUL A. SOUDERS



8. Turn to page 50 of the textbook and answer exercise 1 of "Discussing the Ideas."

9. Turn to page 53 of the textbook and answer "Communicating the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 104–105.

Follow-up Activities

This module dealt with Chapter 1: Measurement in the *Addison-Wesley Applied Mathematics 10 Source Book.*

Turn to page 56 of the textbook and read the chart in "What Do I Need To Know?" Notice the skills and concepts that were developed in this module. Also, read the important results and formulas that you discovered.

Note: You have been using A for surface area; however, it is also acceptable to use S for surface area, as is done in the chart.

Turn to pages 57 and 58 of the textbook.

1. Answer exercises 2 and 3 of Part B of "What Should I Be Able To Do?" **Note:** The second measurement in exercise 3 should read 21.23 mm, not 21.232 mm.

2. Answer exercises 5 and 6 of Part B of "What Should I Be Able To Do?" **Note:** In exercise 5, only the walls will be painted and only one coat of paint is required. Determine the cost of painting the room as well as the number of cans of paint. In exercise 6, round each answer to the nearest whole number.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, page 106–108.

If you had difficulties understanding the skills and concepts in Module 1, Measurement, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

In this module, you performed unit conversions within the metric system of measurement and within the imperial system of measurement.

Many sites on the Internet have conversion calculators. The following sites may be useful to you:

- **http://www.french-property.com/ref/convert.htm**
- **http://www.convertit.com/**

Copy and complete the following tables by using a conversion calculator on the Internet.

1.

Feet	3		4	
Inches		20		70

2.

Yards	8		14	
Feet		28		17

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, page 108.

ENRICHMENT

In this module, you performed unit conversions within the metric and imperial systems of measurement. In this Enrichment activity, you will make conversions between the metric and imperial systems of measurement.

Turn to page 9 of the textbook and read the first paragraph.

1. Gaylene biked 1500 m. Raymond biked 1 mile. Use the table on page 9 of the textbook to decide who biked the longer distance.

Compare your response with the suggested answer in the Appendix, Follow-up Activities: Enrichment, page 108.

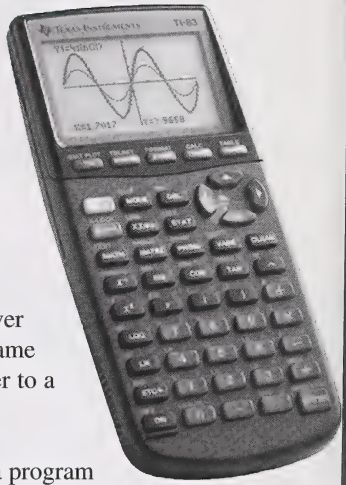
Many sites on the Internet have conversion calculators. The following sites may be useful to you:

- **http://www.french-property.com/ref/convert.htm**
- **http:///www.convertit.com/**

2. Turn to page 20 of the textbook and answer exercises 2, 3, and 4.b. of "Exercises: Checking Your Skills." Use a conversion calculator on the Internet and round each answer to two decimal places.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, page 109.

You should have access to the TI-Graph Link kit (new versions of the TI-83 graphing calculator include this kit; the kit is also available from LRDC). The TI-Graph Link kit includes cables, computer adapters, manuals, software, and sample programs; the TI-Graph Link kit allows TI-graphing-calculator users to transfer files between the calculator and the computer and/or between calculators.



Turn to pages 397 and 398 of the textbook and work through "Utility 4: Linking TI-83 Calculators" to discover how to transfer files between calculators. You use the same procedure when downloading programs from a computer to a TI-83 calculator.

If you have access to the TI-Graph Link kit, download a program called CONVERT from the following site on the Internet:

http://www.awl.com/canada/school/math/ml_app.htm

The CONVERT program allows TI-83 users to perform measurement conversions within and between the measurement systems.

Turn to page 399 of the textbook and work through "Utility 5: Running a Program on a TI-83" to discover how to run a program on the TI-83 calculator.

3. Turn to pages 16 and 17 of the textbook and answer exercises 1, 2, and 3 of "Investigation 2: Using Technology." If you have the CONVERT program on your graphing calculator, use it. If not, use a conversion calculator on the Internet.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, page 109.

When you make measurement conversions **within one** measurement system, the conversions are **exact**. For example, 1 foot = 12 inches.

When you make measurement conversions **between two** measurement systems, the conversions are **approximate**. For example, 1 inch $\doteq 2.54$ cm.

Many recipes require measurement conversions between two measurement systems.

4. Examine the following list of metric cooking units and the imperial units that they replace. How do you think the equivalent measurements were made for the given cooking units?

Metric Measure	Equivalent Imperial Measure
25 mL	2 tablespoons
15 mL	1 tablespoon
5 mL	1 teaspoon
2 mL	$\frac{1}{2}$ of a teaspoon
1 mL	$\frac{1}{4}$ of a teaspoon

Compare your response with the suggested answer in the Appendix, Follow-up Activities: Enrichment, page 109.

MODULE PROJECT

THE DREAM ROOM

Completing the Project

At the beginning of this module you previewed the requirements for your module project. By now, you should have completed your research for the module project, the Dream Room. You now are ready to complete the final phase of the project.

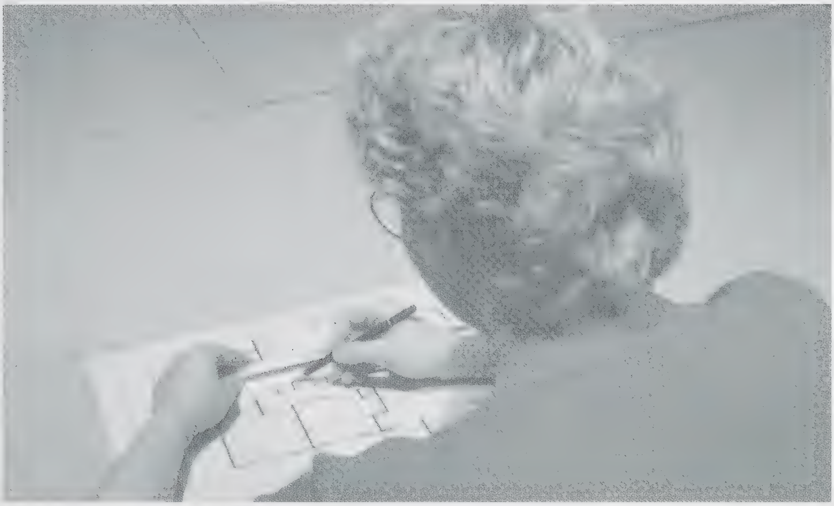
For this project, you must make an accurate floor plan of your room. You can do this by making a scale drawing on graph paper.

1. Turn to page 59 of the textbook and answer exercise 10 of Part C of "What Should I Be Able To Do?"

   Note: The photograph of the student work on page 59 of the textbook has been reduced to fit the page. In reality, each square on the graph paper is 0.5 cm by 0.5 cm.

Compare your response with the suggested answer in the Appendix, Module Project, page 110.

Turn to page 28 of the textbook and read “Planning Your Dream Room.” Follow the directions in the bulleted list to make the first drafts of the plan for your dream room and give an estimate of costs.



Note: You should not work in the space provided in the Project Booklet yet, as you may revise your plans as you work on the project. However, keep copies of your description, sketch(es), scale drawing(s), and calculations in the project section of your mathematics binder.

For this module project, you must make an estimate of the costs.

2. Turn to page 60 of the textbook and answer exercise 11 of Part C of “What Should I Be Able To Do?”

Compare your response with the suggested answer in the Appendix, Module Project, page 111.

Module Project

When you are satisfied with your plans, take out the project booklet that accompanies this Student Module Booklet and complete the module project, The Dream Room.

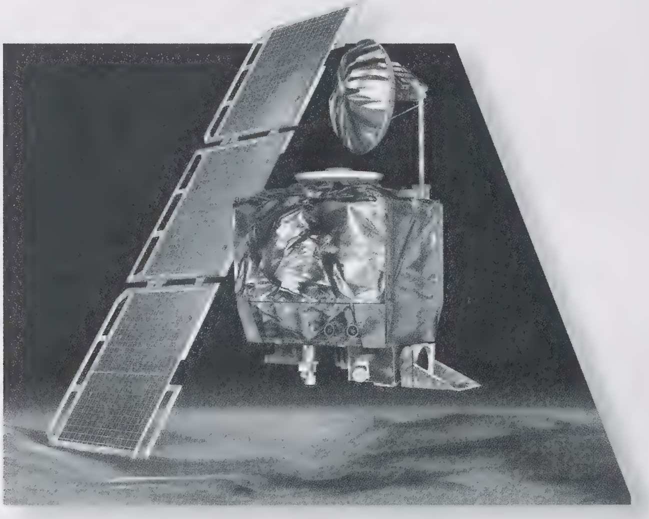
Submit your completed Module 1 Project Booklet to your teacher.

Module Summary

In this module, you used measuring devices to take direct measurements and you made estimates. You calculated indirect measurements, such as perimeter, area, and volume. You then solved problems involving measurements.

Taking measurements and making estimates are very important skills. Making errors can have costly results, as the newspaper article that you read at the beginning of this module illustrates.

Costs cannot only put lives at risk, but can involve large sums of money. Did you know that a metric mistake caused NASA to lose a $125-million Mars orbiter? (If you have access to the Internet, you may wish to discover more details about this costly error. Use search terms such as *Mars orbiter*.)



NASA

NASA's Climate Orbiter was lost September 23, 1999.

Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 1 Assignment Booklet to your teacher.

Applied Mathematics 10

APPENDIX

GLOSSARY

Accuracy: how close a measurement is to the true or accepted value

English Customary System of Measurement: the system of measurement, commonly called the **imperial system**, that uses units such as yards, pounds, and quarts

International System of Units (SI): the measurement system, commonly called the **metric system**, that uses units such as metres, grams, and litres

Precision: how close together a group of measurements actually are to each other; the smallest division on the scale of a measuring device

Tolerance: the permitted amount of variation from a specified measure

SUGGESTED ANSWERS

Activity 1: Taking and Estimating Measurements

1. A $\frac{1}{8}$ of an inch B $\frac{1}{2}$ of an inch C $\frac{3}{4}$ of an inch D $1\frac{7}{8}$ inches E $2\frac{5}{8}$ inches

F $\frac{9}{16}$ of an inch G $\frac{13}{16}$ of an inch H $1\frac{1}{2}$ inches I $1\frac{15}{16}$ inches J $2\frac{11}{16}$ inches

2. A 1 foot $\frac{11}{16}$ inches B 1 foot $1\frac{1}{4}$ inches

C 1 foot $2\frac{9}{16}$ inches D 1 foot $3\frac{1}{16}$ inches

There are several ways of reporting imperial measurements. For example, 1 foot $\frac{11}{16}$ inches may be written as 1 ft. $\frac{11}{16}$ in. or 1 ' $\frac{11}{16}$ ".

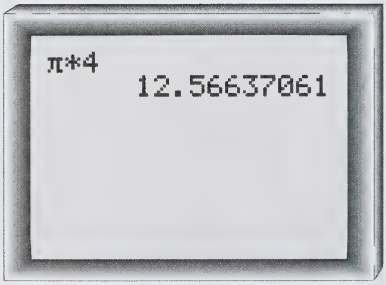
3. a. $C = \pi d$

$= \pi(4)$

$\doteq 13$

The trundle wheel covers approximately 13 inches.

2nd [π] × 4 ENTER



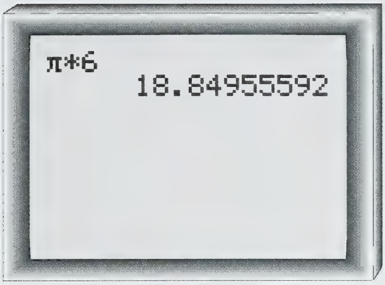
```
π*4
          12.56637061
```

b. $C = \pi d$

$= \pi(6)$

$\doteq 19$

2nd [π] × 6 ENTER



```
π*6
          18.84955592
```

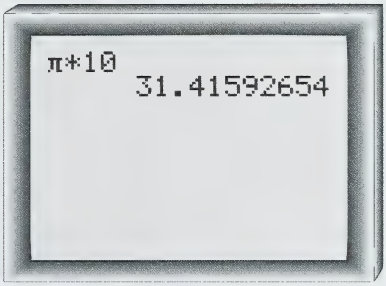
The trundle wheel covers approximately 19 inches.

c. $C = \pi d$

$= \pi(10)$

$\doteq 31$

2nd [π] × 1 0 ENTER



```
π*10
          31.41592654
```

The trundle wheel covers approximately 31 inches.

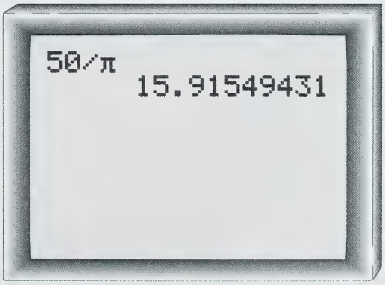
4. $C = \pi d$

$50 = \pi d$

$\frac{50}{\pi} = d$

$d \doteq 16$

5 0 ÷ 2nd [π] ENTER



```
50/π
          15.91549431
```

Rounded to the nearest whole number, the diameter is 16 cm.

Activity 1 (continued)

5. The digital display on a trundle wheel is helpful because it eliminates the need of manually counting the number of revolutions that the wheel turns and calculating the distance covered. It is quite easy to get distracted and lose count when manually counting long distances.

6. a. **Metric Scale on Vernier Caliper #1, p. 391**

The scale on the main arm shows that the coarse reading is 6.8 cm.

The vernier scale shows that 0.01 cm must be added to the coarse reading.

$6.8 + 0.01 = 6.81$ **or** $$\begin{array}{r} 6.8 \\ +\,0.01 \\ \hline 6.81 \end{array}$$

The metric reading is 6.81 cm.

Metric Scale on Vernier Caliper #2, p. 391

The scale on the main arm shows that the coarse reading is 0.0 cm.

The vernier scale shows that 0.04 cm must be added to the coarse reading.

$0.0 + 0.04 = 0.04$ **or** $$\begin{array}{r} 0.0 \\ +\,0.04 \\ \hline 0.04 \end{array}$$

The metric reading is 0.04 cm.

b. **Imperial Scale on Vernier Caliper #1, p. 391**

The scale on the main arm shows that the coarse reading is 2.65 inches.

The vernier scale shows that 0.03 of an inch must be added to the coarse reading.

$2.65 + 0.03 = 2.68$ **or** $$\begin{array}{r} 2.65 \\ +\,0.03 \\ \hline 2.68 \end{array}$$

The imperial reading is 2.68 inches.

Imperial Scale on Vernier Caliper #2, p. 391

The scale on the main arm shows that the coarse reading is 0.00 inches.

The vernier scale shows that 0.01 of an inch must be added to the coarse reading.

$0.00 + 0.01 = 0.01$ **or** $\begin{array}{r} 0.00 \\ +\,0.01 \\ \hline 0.01 \end{array}$

The imperial reading is 0.01 of an inch.

7. **a.** The scale on the main arm shows that the coarse reading is $2\frac{13}{16}$ inches.

The vernier scale shows that $\frac{4}{128} = \frac{1}{32}$ of an inch must be added to the coarse reading.

$$2\frac{13}{16} + \frac{1}{32} = 2\frac{26}{32} + \frac{1}{32}$$
$$= 2\frac{27}{32}$$

or

$$\begin{array}{rr} 2\dfrac{13}{16} = & 2\dfrac{26}{32} \\ +\ \dfrac{1}{32} = & \dfrac{1}{32} \\ \hline & 2\dfrac{27}{32} \end{array}$$

The imperial reading is $2\frac{27}{32}$ inches.

b. The scale on the main arm shows that the coarse reading is $3\frac{15}{16}$ inches.

The vernier scale shows that $\frac{5}{128}$ of an inch must be added to the coarse reading.

$$3\frac{15}{16} + \frac{5}{128} = 3\frac{120}{128} + \frac{5}{128}$$
$$= 3\frac{125}{128}$$

or

$$\begin{array}{rr} 3\dfrac{15}{16} = & 3\dfrac{120}{128} \\ +\ \dfrac{5}{128} = & \dfrac{5}{128} \\ \hline & 3\dfrac{125}{128} \end{array}$$

The imperial reading is $3\frac{125}{128}$ inches.

Activity 1 (continued)

c. The scale on the main arm shows that the coarse reading is $1\frac{11}{16}$ inches.

The vernier scale shows that $\frac{0}{128}=0$ of an inch must be added to the coarse reading.

The imperial reading is $1\frac{11}{16}$ inches.

d. The scale on the main arm shows that the coarse reading is $\frac{13}{16}$ of an inch.

The vernier scale shows that $\frac{1}{128}$ of an inch must be added to the coarse reading.

$$\begin{aligned}\frac{13}{16}+\frac{1}{128}&=\frac{104}{128}+\frac{1}{128}\\&=\frac{105}{128}\end{aligned}$$

$$\begin{array}{rr}& \frac{13}{16}=\frac{104}{128}\\ + & \frac{1}{128}=\frac{1}{128}\\ \hline & \frac{105}{128}\end{array}$$

The imperial reading is $\frac{105}{128}$ of an inch.

8. A digital display is helpful because it eliminates the need to take a coarse reading, take a vernier reading, and then add the two readings together.

9. **Micrometer #1 reading, p. 395**

The coarse reading is 13.0 mm. The barrel scale indicates that 0.00 mm must be added to the coarse reading.

$13.0+0.00=13.00$

The micrometer reading is 13.00 mm.

Micrometer #2 reading, p. 395

The coarse reading is 3.5 mm. The barrel scale indicates that 0.30 mm must be added to the coarse reading.

$3.5+0.30=3.80$

The micrometer reading is 3.80 mm.

10. **a.** Subtract 0.01 mm from the reading.

b. Add 0.01 mm to the reading.

11. Answers will vary. Comments are given.

Were you surprised at the number and variety of measuring devices people use every day? Did you speak with any of the following?

- a carpenter
- a homemaker
- a nurse
- a mechanic
- a construction estimator

Be sure your answer is in the form of a short report.

Activity 2: Working with Measurements

1. Textbook exercises 1 and 2 of "Investigation 2: Visualizing Measurement," p. 11

1. **a.** feet and inches (for example, 5 feet 6 inches)

b. inches

c. miles

d. feet and inches (for example, 30 feet 6 inches)

or

yards and inches (for example, 3 yards 6 inches)

e. inches

f. inches

Activity 2 (continued)

2. **a.** centimetres (for example, 178 cm) **or** metres (for example, 1.78 m)
   b. centimetres **or** millimetres
   c. kilometres
   d. metres
   e. centimetres
   f. centimetres **or** millimetres

2. Textbook exercises 1 and 3 of "Discussing the Ideas," p. 13

1. Answers will vary. Sample answers are given.

   a. The width of two fingers is approximately 1 inch.
   b. A "baby step," which is a bit more than the length of your foot, is approximately 1 foot.
   c. A "giant step," which is a long stride, is approximately the length of 1 yard.

3. You could count the number of "baby steps" or "giant steps" from your desk to the front of the classroom. Distances at home could be estimated in the same way (you could estimate the distance between your sofa and the TV, for instance). The number of "baby steps" would approximately equal the number of feet; the number of "giant steps" would approximately equal the number of yards.

   How did your estimate compare to others?

3.

Yards	1	2	6	$3\frac{1}{3}$	10	$8\frac{1}{3}$
Feet	3	6	18	10	30	25

Inches	12	36	48	18	64	108
Feet	1	3	4	$1\frac{1}{2}$	$5\frac{1}{3}$	9

4. Textbook exercise 2 of "Discussing the Ideas," p. 19

2. If you were converting a measurement from a smaller unit to a larger unit, you would expect to have fewer units.

   For example, $200\ \text{cm} = 2\ \text{m}$, and $24\ \text{inches} = 2\ \text{feet}$.

5. **Step 1:** Change the units to feet. **Note:** In this problem, it is easier to use decimals.

$$\begin{aligned} \text{12 feet 6 inches } &= 12\frac{6}{12}\text{ feet} \\ &= 12\frac{1}{2}\text{ feet} \\ &= 12.5\text{ feet} \end{aligned} \qquad \begin{aligned} \text{10 feet 3 inches } &= 10\frac{3}{12}\text{ feet} \\ &= 10\frac{1}{4}\text{ feet} \\ &= 10.25\text{ feet} \end{aligned}$$

Step 2: Find the area of the ceiling.

$$\begin{aligned} A &= \ell w \\ &= 12.5 \times 10.25 \\ &= 128.125 \end{aligned}$$

The area of the ceiling is 128.125 square feet.

Step 3: Find the total area to be covered. Because 2 coats of paint are needed, multiply the area of the ceiling by 2.

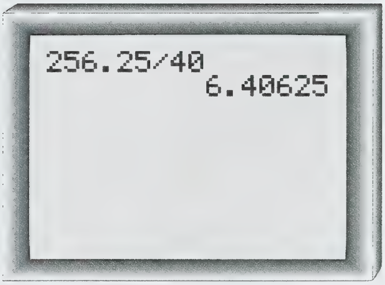
$$2 \times 128.125 = 256.25$$

The area to be covered is 256.25 square feet.

Step 4: Determine the number of cans of paint needed. **Note:** Because Andrea cannot buy part of a can of paint, round the answer up to the nearest whole number.

$$256.25 \div 40 \doteq 7$$

Andrea will need 7 cans of paint.



```
256.25/40
             6.40625
```

6. **Step 1:** Rewrite the measurements in feet. **Note:** In this problem, it is easier to use fractions.

$$\begin{aligned} \text{13 feet 6 inches } &= 13\frac{6}{12}\text{ feet} \\ &= 13\frac{1}{2}\text{ feet} \end{aligned} \qquad \begin{aligned} \text{11 feet 4 inches } &= 11\frac{4}{12}\text{ feet} \\ &= 11\frac{1}{3}\text{ feet} \end{aligned}$$

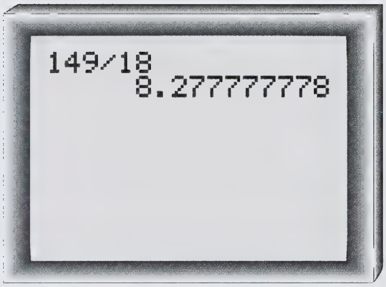
Activity 2 (continued)

Step 2: Find the perimeter.

$$\begin{aligned} P &= \ell + \ell + w + w \\ &= 13\tfrac{1}{2} + 13\tfrac{1}{2} + 11\tfrac{1}{3} + 11\tfrac{1}{3} \\ &= 27 + 22\tfrac{2}{3} \\ &= 49\tfrac{2}{3} \end{aligned}$$

The perimeter is $49\frac{2}{3}$ feet.

Step 3: Determine the number of panels needed.
Note: Because Juan cannot buy part of a panel, round the answer up to the nearest whole number.



```
149/18
          8.277777778
```

$$\begin{aligned} 49\tfrac{2}{3} \div 6 &= \frac{149}{3} \times \frac{1}{6} \\ &= \frac{149}{18} \\ &\doteq 9 \end{aligned}$$

Juan will need 9 panels.

7. Because William's selection has a 5-inch repeat pattern, the usable yield is 25 square feet (according to the usable yield table in Activity 2).

Step 1: Find the required number of single rolls.

$420 \div 25 \doteq 17$ ← rounded up to the nearest whole number

Step 2: Find the required number of double rolls. **Hint:** Because you cannot buy part of roll, you must round up to the nearest whole number.

$17 \div 2 \doteq 9$ ← rounded up to the nearest whole number

William will need to purchase 9 double rolls of wallpaper.

8. Textbook exercise 2 of "Investigation 4: Working with Area," p. 12

2. The side length of the square is 100 cm. **Note:** 1 m = 100 cm.

There are 10 000 cm^2 in 1 m^2. **Note:** $100 \times 100 = 10\ 000$

9. Textbook exercise 4 of "Investigation 4: Working with Area," p. 12

4. The investigation shows the following:

- There are more than 9 square feet in 1 m^2. **Note:** If you piece together the extra newspaper, you will discover that there is a little less than 11 square feet in 1 m^2.
- There is a little more than 1 square yard in 1 m^2.
- There are exactly 9 square feet in 1 square yard.

10. Textbook exercise 4.a. of "Exercises: Checking Your Skills," p. 20

4. a. Step 1: Determine the number of square yards in 1 square foot.

$$1 \text{ square yard} = 9 \text{ square feet}$$
$$\therefore\ 1 \text{ square foot} = \frac{1}{9} \text{ square yard}$$

Step 2: Calculate the area in square yards. Round to the nearest square yard.

$$\frac{1}{9} \times 5.2 \text{ million} = \frac{5\ 200\ 000}{9}$$
$$\doteq 577\ 778$$

5.2 million = 5 200 000

West Edmonton Mall covers an area of approximately 577 778 square yards.

11. Textbook question "Communicating the Ideas," p. 13

Answers will vary. Comments are given.

Perhaps you were able to use wall or floor tiles to estimate small measurements, such as inches or feet. Perhaps you used travel time as an estimating tool for large measurements, such as kilometres. You could also relate imperial units to metric units, which you can already visualize. How accurate were your strategies?

Activity 3: Precision and Accuracy

1. **Textbook exercise 3 of "Exercises: Checking Your Skills," p. 27**

   3. Diagram c. represents the best precision and accuracy. The four shots are close together (precision) and near the bull's-eye (accuracy).

2. **a.** The precision of the ruler is 0.1 of an inch.

   b. The uncertainty of the measurement is ± 0.05 of an inch.

   c. The true measurement of the book is between 8.55 inches and 8.65 inches.

3. **a.** The precision of the ruler is $\frac{1}{8}$ of an inch.

   b. The uncertainty of the measurement is $\pm\frac{1}{16}$ of an inch.

   c. The true measurement is between 22 feet $\frac{9}{16}$ inches and 22 feet $\frac{11}{16}$ inches.

4. **a.** The uncertainty of the measurement is ± 0.0005 of an inch.

   b. The true measurement is between 6.1235 inches and 6.1245 inches.

5. No, the answer is not reasonable. It does not seem reasonable to obtain an answer that is precise to 4 decimal places when the original measurements were only precise to 2 decimal places. **Note:** Turn to page 24 of the textbook and read the more detailed explanation at the top of the page.

6. **a.** $4 - 0.1 = 3.9$

   The minimum acceptable diameter of the shaft is 3.9 mm.

   b.

Length	Width	Height
$11 + \frac{1}{8} = 11\frac{1}{8}$	$4\frac{1}{4} + \frac{1}{8} = 4\frac{2}{8} + \frac{1}{8}$ $= 4\frac{3}{8}$	$5\frac{3}{4} + \frac{1}{8} = 5\frac{6}{8} + \frac{1}{8}$ $= 5\frac{7}{8}$

   The maximum acceptable length is $11\frac{1}{8}$ inches; the maximum acceptable width is $4\frac{3}{8}$ inches; and the maximum acceptable height is $5\frac{7}{8}$ inches.

7. $10\% = 0.10 \text{ or } 0.1$

$$\begin{aligned} 10\% \text{ of } 120 &= 0.1 \times 120 \\ &= 12 \end{aligned}$$

$$120 - 12 = 108$$
$$120 + 12 = 132$$

The motor is designed to run between 108 volts and 132 volts.

8. **Textbook question "Communicating the Ideas," p. 27**

Accuracy is how close a measurement is to the true or accepted value. Precision has two meanings. Precision indicates how close together a group of measurements actually are to each other. Precision also indicates the unit of the measure.

When measuring very small objects, a high degree of accuracy and precision are required. For example, if the diameter of a strand of hair is measured with a ruler whose smallest division is 1 mm, the measure would not be accurate nor precise.

On the other hand, if you are measuring your height, it is not necessary to have more precision than 1 mm.

Activity 4: Volume and Surface Area of a Sphere

1. **Textbook exercise 1 of "Discussing the Ideas," p. 33**

   1. It is easy to make measurement errors. By measuring the diameter 5 times and discarding the highest and lowest measurement, the remaining measurements should be closer to the true diameter.

2. **Textbook exercises 4, 5, and 6 of "Investigation: The Volume of a Sphere," p. 31**

   4. The volume of the water that was made to overflow is the water that has been displaced by the ball. This volume is equal to the volume of the ball.

   5. At the end of textbook exercise 3, $\frac{2}{3}$ of the can was filled with water.

   6. You could conclude that the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder.

3. **Textbook exercise 2 of "Discussing the Ideas," p. 33**

   2. The volume of the ball was found by determining the amount of water that it displaces. To prevent the ball from absorbing any of the water and thereby causing a measurement error, the tennis ball was first soaked in water.

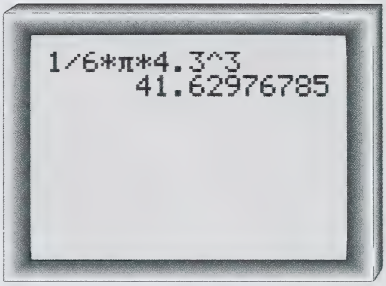
4. Textbook exercises 1 to 4 of "Exercises: Checking Your Skills," p. 34

Note: Rules for rounding in Applied Mathematics 10 are fairly flexible. Therefore, your answers may be rounded to a different place value than the suggested answers given here. In your calculations, you should be using the π key on your calculator instead of a rounded value of π.

1. a.

$$\begin{aligned} V &= \frac{1}{6}\pi d^3 \\ &= \frac{1}{6}\pi (4.3)^3 \\ &\doteq 42 \end{aligned}$$

The volume of the golf ball is approximately 42 cm^3.

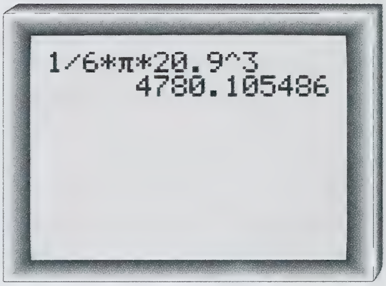


```
1/6*π*4.3^3
            41.62976785
```

b.

$$\begin{aligned} V &= \frac{1}{6}\pi d^3 \\ &= \frac{1}{6}\pi (20.9)^3 \\ &\doteq 4780 \end{aligned}$$

The volume of the volleyball is approximately 4780 cm^3.

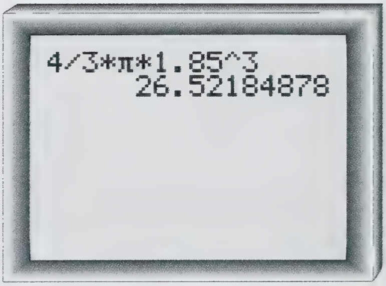


```
1/6*π*20.9^3
            4780.105486
```

c.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi (1.85)^3 \\ &\doteq 26.5 \end{aligned}$$

The volume of the tennis ball is approximately 26.5 cm^3.

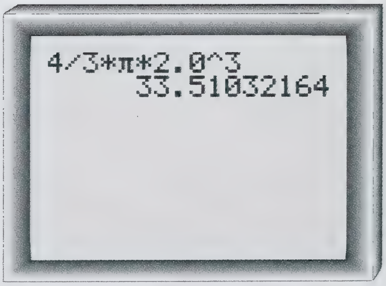


```
4/3*π*1.85^3
            26.52184878
```

d.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi (2.0)^3 \\ &\doteq 34 \end{aligned}$$

The volume of the squash ball is approximately 34 cm^3.

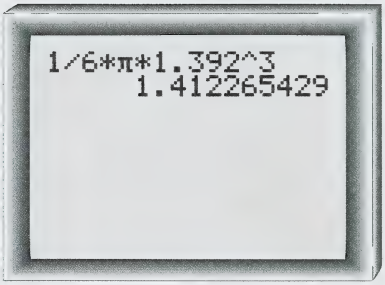


```
4/3*π*2.0^3
            33.51032164
```

2. $$\begin{aligned} V &= \frac{1}{6}\pi d^3 \\ &= \frac{1}{6}\pi\left(1.392\times10^6\right)^3 \\ &= \frac{1}{6}\pi\times(1.392)^3\times10^{18} \\ &\doteq 1.412\times10^8 \end{aligned}$$

You write 1.392 million in scientific notation as 1.392×10^6.

← The Power of a Product Law was used.



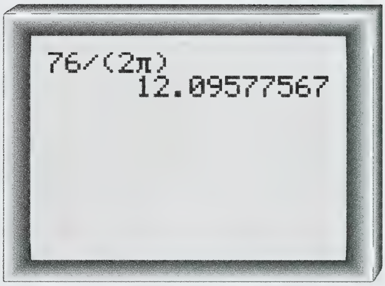
```
1/6*π*1.392^3
       1.412265429
```

The volume of the Sun is approximately 1.412×10^{18} km^3.

Note: There is more information on the laws of exponents in the student reference section in the textbook; see page 422.

3. **a.** $$\begin{aligned} C &= 2\pi r \\ 76 &= 2\pi r \\ r &= \frac{76}{2\pi} \\ &\doteq 12 \end{aligned}$$

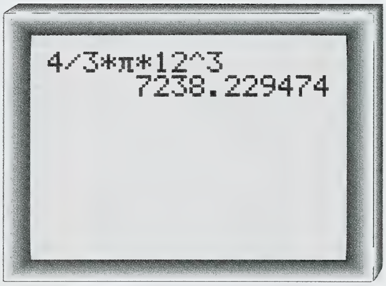
The radius of the basketball is approximately 12 cm.



```
76/(2π)
       12.09577567
```

b. $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(12)^3 \\ &\doteq 7200 \end{aligned}$$

The volume of the basketball is approximately 7200 cm^3.



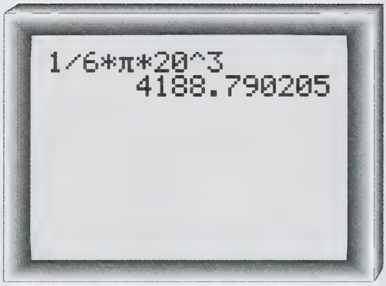
```
4/3*π*12^3
       7238.229474
```

Activity 4 (continued)

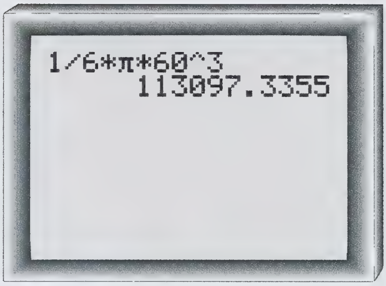
4. Step 1: Calculate the balloon's original volume and new volume.

$$\text{original volume} = \frac{1}{6}\pi d^3 = \frac{1}{6}\pi(20)^3 \doteq 4200$$

$$\text{new volume} = \frac{1}{6}\pi d^3 = \frac{1}{6}\pi(60)^3 \doteq 110\ 000$$



```
1/6*π*20^3
          4188.790205
```



```
1/6*π*60^3
          113097.3355
```

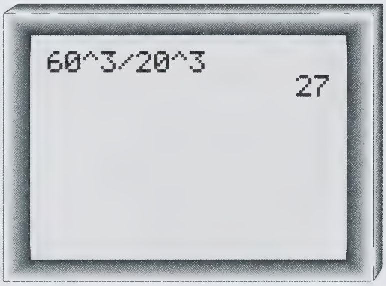
The original volume was approximately $4200\ \text{cm}^3$, and the new volume is approximately $110\ 000\ \text{cm}^3$.

Note: Do not use these rounded answers in the second part of the question.

Step 2: Calculate how many times the volume has increased.

$$\frac{\text{new volume}}{\text{original volume}} = \frac{\cancel{\frac{1}{6}}\cancel{\pi}(60)^3}{\cancel{\frac{1}{6}}\cancel{\pi}(20)^3} = \frac{60^3}{20^3} = 27$$

The volume increased by 27 times.



```
60^3/20^3
                   27
```

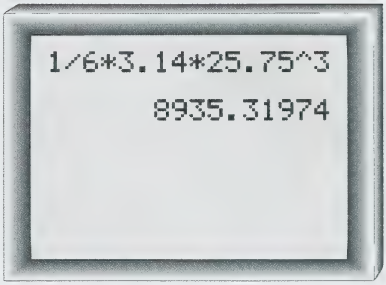
5. Textbook question "Communicating the Ideas," p. 35

Answers will vary. A sample response is given using a sphere with a diameter of 25.75 cm.

Using the Rounded Value of π

$$\begin{aligned} V &= \frac{1}{6}\pi d^3 \\ &\doteq \frac{1}{6}(3.14)(25.75)^3 \\ &\doteq 8935 \end{aligned}$$

The volume is approximately 8935 cm^3.

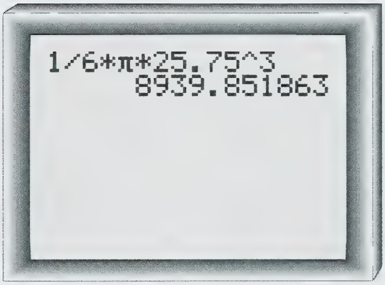


```
1/6*3.14*25.75^3
        8935.31974
```

Using the π Key

$$\begin{aligned} V &= \frac{1}{6}\pi d^3 \\ &= \frac{1}{6}\pi(25.75)^3 \\ &\doteq 8940 \end{aligned}$$

The volume is approximately 8940 cm^3.



```
1/6*π*25.75^3
        8939.851863
```

Using the rounded value of π in this example produces a volume that is approximately 5 cm^3 less than when using the π key.

6. Textbook exercise 1 of "Discussing the Ideas," p. 38

1. Strictly speaking, an orange is not a perfect sphere. It may be slightly flattened and not perfectly symmetrical. However, an orange is very close to being a perfect sphere. You can tell this because the measurements of the diameter taken in 5 different places are very close in size.

7. Textbook exercises 2 and 3 of "Discussing the Ideas," p. 38

2. The measurement of the diameter is more accurate than the measurement of the surface area. In other words, there was less of a chance for measurement error when the diameter was measured than when the surface area was measured.

The reason the measurement of the diameter has less uncertainty is because it was measured with a vernier caliper. The surface area, on the other hand, was determined by a less precise method—laying the pieces of orange peel on graph paper, pressing the peels down, tracing around the flattened peels, and then estimating the area by counting the full and partial squares.

Activity 4 (continued)

3. You could increase the precision of the measuring device (namely the graph paper) used to find the surface area of the orange by using graph paper that has smaller subdivisions. This, however, would make the area more difficult to determine because more full and partial squares would have to be counted. It may also be more difficult to estimate the areas of partial squares.

8. Textbook exercises 6, 7, and 8 of "Investigation: Surface Area of a Sphere," p. 37

6. Answers will vary depending on the results obtained in exercise 5; however, the ratio should be about 3. In reality, the answer is π.

$$\frac{\text{surface area}}{\text{diameter}^2} = \frac{\pi \cancel{d}^2}{\cancel{d}^2}$$
$$= \pi$$

The decimal approximation of π is 3.141 592 654…

7. $$\frac{A}{d^2} = \pi$$
$$A = \pi d^2$$

8. $$A = \pi d^2$$
$$= \pi (2r)^2$$
$$= \pi 4r^2$$
$$= 4\pi r^2$$

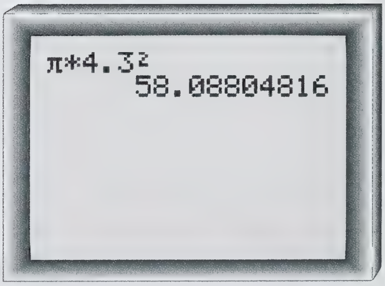
9. Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 39

For exercise 1, your answers must be rounded to the nearest whole number.

For exercise 2, your answer may be rounded to a different place value than the suggested answer given. Just remember to round to a reasonable place value.

1. a. $$A = \pi d^2$$
$$= \pi (4.3)^2$$
$$\doteq 58$$

The surface area of the golf ball is approximately 58 cm^2.



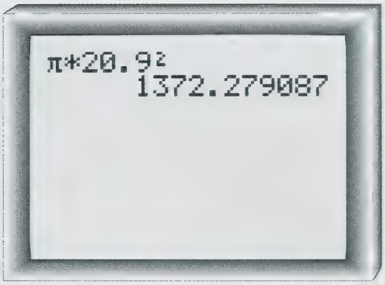
```
π*4.3²
          58.08804816
```

b. $A = \pi d^2$

$= \pi (20.9)^2$

$\doteq 1370$

The surface area of the volleyball is approximately $1370\ \text{cm}^2$.



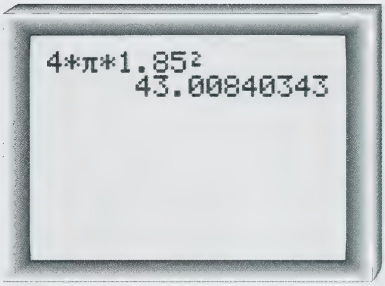
```
π*20.9²
         1372.279087
```

c. $A = 4\pi r^2$

$= 4\pi (1.85)^2$

$\doteq 43.0$

The surface area of the tennis ball is approximately $43.0\ \text{cm}^2$.



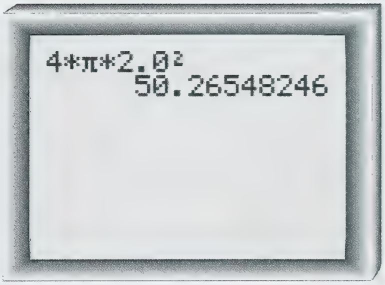
```
4*π*1.85²
         43.00840343
```

d. $A = 4\pi r^2$

$= 4\pi (2.0)^2$

$\doteq 50$

The surface area of the squash ball is approximately $50\ \text{cm}^2$.



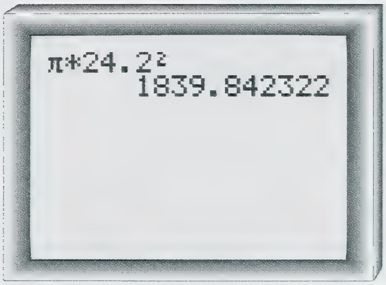
```
4*π*2.0²
         50.26548246
```

2. $A = \pi d^2$

$= \pi (24.2)^2$

$\doteq 1840$

The surface area of the basketball is approximately $1840\ \text{cm}^2$.



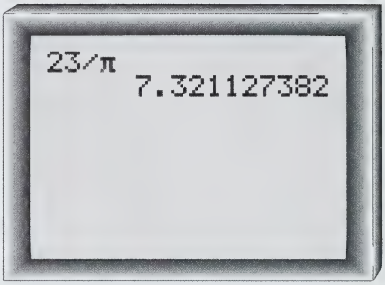
```
π*24.2²
         1839.842322
```

10. Textbook exercises 3, 4, and 5 of "Exercises: Checking Your Skills," p. 39

Note: For exercises 3 to 5, your answers may be rounded to different place values than the suggested answers given. Just remember to round to a reasonable place value.

3. **Step 1:** Determine the diameter.

2 3 ÷ 2nd [π] ENTER



```
23/π
          7.321127382
```

$$C = \pi d$$
$$23 = \pi d$$
$$\frac{23}{\pi} = d$$
$$7.3 \doteq d$$

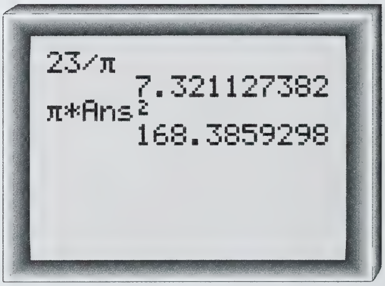
The diameter of the baseball is approximately 7.3 cm.

Note: This is a rounded answer. Do **not** use the rounded answer in the calculations in Step 2. Instead, keep the calculator running and use the answer key in Step 2.

Step 2: Determine the surface area.

2nd [π] × 2nd [ANS]

x^2 ENTER



```
23/π
          7.321127382
π*Ans²
          168.3859298
```

$$A = \pi d^2$$
$$\doteq 168$$

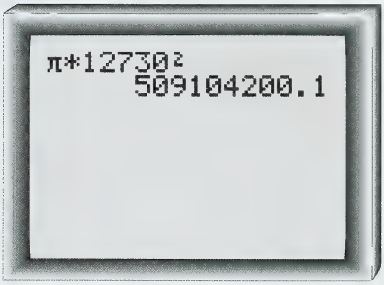
The surface area of the baseball is approximately 168 cm^2.

4. **a.** $A = \pi d^2$

$\doteq \pi (12\,730)^2$

$\doteq 509\,100\,000$ **or** 5.091×10^8

2nd [π] × 1 2 7 3 0 x^2 ENTER



```
π*12730²
         509104200.1
```

The surface area of Earth is approximately $509\,100\,000\ \text{km}^2$ **or** $5.091 \times 10^8\ \text{km}^2$.

Note: This is a rounded answer. Do **not** use the rounded answer in Step 2 of exercise 4.b. Instead, keep the calculator running and use the answer key in exercise 4.b.

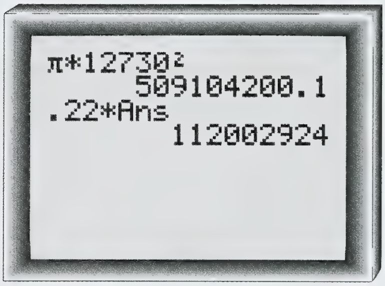
b. **Step 1:** Determine the percent of land.

$100\% - 78\% \doteq 22\%$

If approximately 78% of land is covered with water, then approximately 22% of Earth's surface is land.

Step 2: Find the area of land on Earth's surface.

1 2 2 × 2nd [ANS] ENTER



```
π*12730²
         509104200.1
.22*Ans
           112002924
```

Note: Because $22\% = 0.22$, multiply the unrounded answer in Step 1 by 0.22.

The area of land on Earth's surface is approximately $110\,000\,000\ \text{km}^2$ **or** $1.1 \times 10^8\ \text{km}^2$.

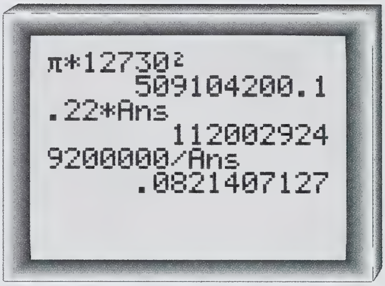
Note: This is a rounded answer. Do **not** use the rounded answer in exercise 4.c. Instead, keep the calculator running and use the answer key in exercise 4.c.

Activity 4 (continued)

c. **Step 1:** Calculate the decimal fraction that represents Canada's land area divided by the total land area of Earth. **Note:** 9.2 million = 9 200 000

Canada's land area is approximately 0.08 of the total land area of Earth.

9 2 0 0 0 0 0 ÷ 2nd [ANS] × 1 0 0



```
π*12730²
         509104200.1
.22*Ans
           112002924
9200000/Ans
         .0821407127
```

Step 2: Calculate the percent that is equivalent to 0.08.

$$0.08 = 8\%$$

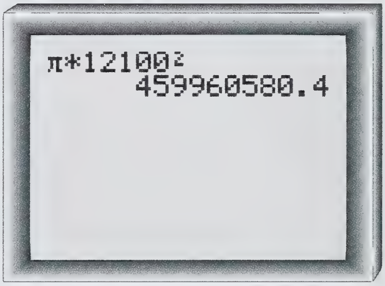
Canada's land area is approximately 8% of the total land area of Earth.

5. **a.** $A = \pi d^2$

$\doteq \pi\left(12\ 100\right)^2$

$\doteq 460\ 000\ 000$ **or** 4.60×10^8

2nd [π] × 1 2 1 0 0 x^2 ENTER



```
π*12100²
         459960580.4
```

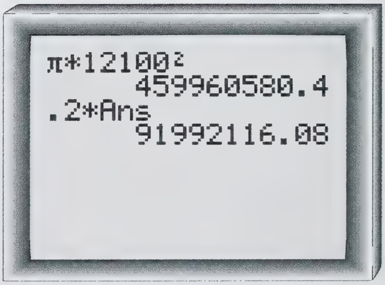
The surface area of Venus is approximately $460\ 000\ 000\ \text{km}^2$ **or** $4.60 \times 10^8\ \text{km}^2$.

Note: Do not clear your calculator. Use the answer key in exercise 5.b.

b. Because $20\% = 0.20$ or 0.2, multiply the unrounded answer in exercise 5.a. by 0.2.

The area of the lowland plains on Venus is about $92\ 000\ 000\ \text{km}^2$ or $9.2 \times 10^8\ \text{km}^2$.

• 2 × 2nd [ANS] ENTER



```
π*12100²
         459960580.4
.2*Ans
         91992116.08
```

11. Textbook question "Communicating the Ideas," p. 40

Answers will vary.

Formulas applying to area will have a degree of 2; formulas applying to volume will have a degree of 3.

A measurement given in square units represents an area; a measurement given in cubic units represents a volume.

Note: You learned about the degree of a polynomial in Mathematics 9. The degree of a term is the sum of the exponents of the variable(s). The degree of a polynomial is the highest degree of its terms.

Examples

$$\begin{aligned} A &= \ell w \\ &= \ell^1 w^1 \longleftarrow \text{degree 2} \end{aligned}$$

$$\begin{aligned} V &= \ell w h \\ &= \ell^1 w^1 h^1 \longleftarrow \text{degree 3} \end{aligned}$$

$$S = 4\pi r^2 \longleftarrow \text{degree 2}$$

$$V = \frac{4}{3}\pi r^3 \longleftarrow \text{degree 3}$$

π is not a variable.

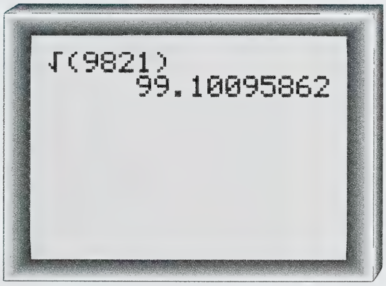
Activity 5: Square Roots and Cube Roots

1. a. Estimated Value

$$\sqrt{9821} = \sqrt{\underset{\substack{\downarrow\\10}}{98}\,\underset{\substack{\downarrow\\0}}{21}}$$

$$\doteq 100$$

Calculated Value



```
√(9821)
          99.10095862
```

$\therefore\ \sqrt{9821} \doteq 99.10$

Activity 5 (continued)

b. Estimated Value

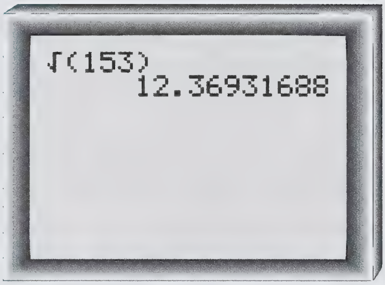
$\sqrt{153} = \sqrt{0153}$

$\downarrow\ \downarrow$

$1\ \ 0$

$\doteq 10$

Calculated Value



```
√(153)
            12.36931688
```

$\therefore\ \sqrt{153} \doteq 12.37$

c. Estimated Value

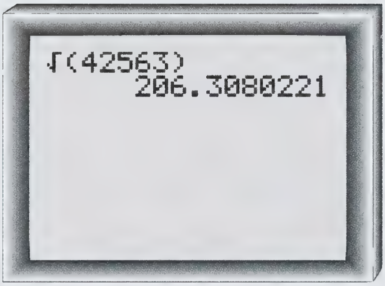
$\sqrt{42\ 563} = \sqrt{042\ 563}$

$\downarrow\ \downarrow\ \downarrow$

$2\ \ 0\ \ 0$

$\doteq 200$

Calculated Value



```
√(42563)
            206.3080221
```

$\therefore\ \sqrt{42\ 563} \doteq 206.31$

d. Estimated Value

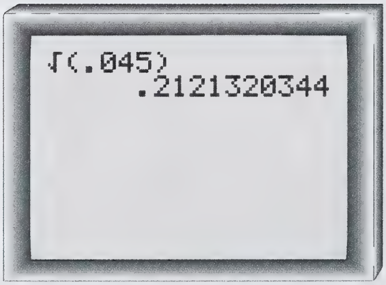
$\sqrt{0.045} = \sqrt{0.0450}$

$\downarrow\ \downarrow$

$.2\ \ 0$

$\doteq 0.20$

Calculated Value



```
√(.045)
            .2121320344
```

$\therefore\ \sqrt{0.045} \doteq 0.21$

e. Estimated Value

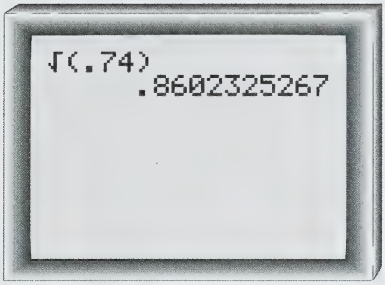
$\sqrt{0.74} = \sqrt{0.74}$

$\downarrow$

$.9$

$\doteq 0.9$

Calculated Value



```
√(.74)
         .8602325267
```

$\therefore\ \sqrt{0.74} \doteq 0.86$

f. Estimated Value

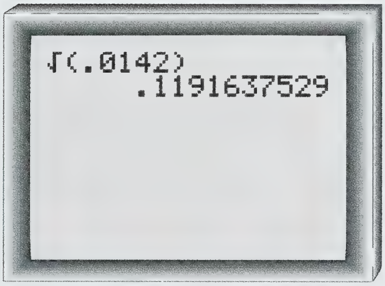
$\sqrt{0.0142} = \sqrt{0.0142}$

$\downarrow\ \downarrow$

$.1\ \ 0$

$\doteq 0.10$

Calculated Value



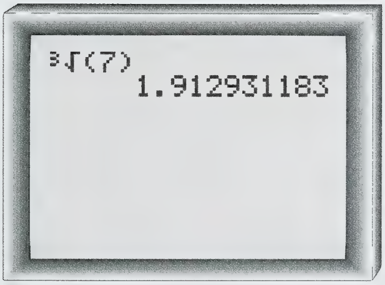
```
√(.0142)
         .1191637529
```

$\therefore\ \sqrt{0.0142} \doteq 0.12$

2. a. Estimated Value

Because $1 \times 1 \times 1 = 1$ and $2 \times 2 \times 2 = 8$, $\sqrt[3]{7}$ is between 1 and 2.

Calculated Value

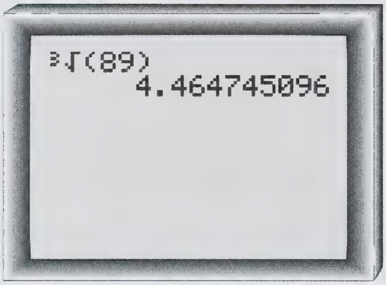


```
³√(7)
         1.912931183
```

$\therefore\ \sqrt[3]{7} \doteq 1.91$

b. **Estimated Value** | **Calculated Value**

Because $4 \times 4 \times 4 = 64$ and $5 \times 5 \times 5 = 125$, $\sqrt[3]{89}$ is between 4 and 5.

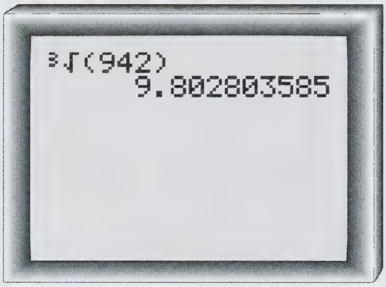


```
³√(89)
        4.464745096
```

$\therefore\ \sqrt[3]{89} \doteq 4.46$

c. **Estimated Value** | **Calculated Value**

Because $9 \times 9 \times 9 = 729$ and $10 \times 10 \times 10 = 1000$, $\sqrt[3]{942}$ is between 9 and 10.



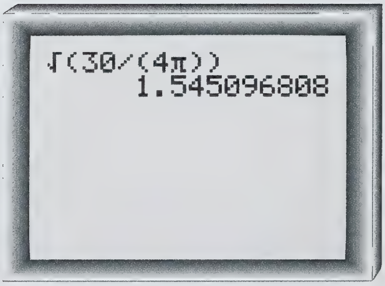
```
³√(942)
        9.802803585
```

$\therefore\ \sqrt[3]{942} \doteq 9.80$

3. Textbook exercises 2, 4, and 7 of "Exercises: Checking Your Skills," p. 44

2.

$$A = 4\pi r^2$$
$$30 = 4\pi r^2$$
$$\frac{30}{4\pi} = r^2$$
$$r = \sqrt{\frac{30}{4\pi}}$$
$$\doteq 1.5$$



```
√(30/(4π))
        1.545096808
```

The radius is about 1.5 cm.

4. $V = \frac{1}{6}\pi d^3$

$40 = \frac{1}{6}\pi d^3$

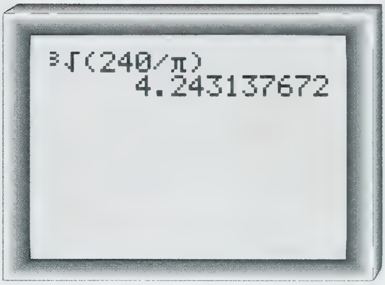
$240 = \pi d^3$

$\frac{240}{\pi} = d^3$

$d = \sqrt[3]{\frac{240}{\pi}}$

$\doteq 4.2$

The diameter is about 4.2 m.



```
³√(240/π)
             4.243137672
```

7. **a.** **Step 1:** Find the new volume.

$24 + 20 = 44$

The new volume is 44 cm^3.

Step 2: $V = \frac{1}{6}\pi d^3$

$44 = \frac{1}{6}\pi d^3$

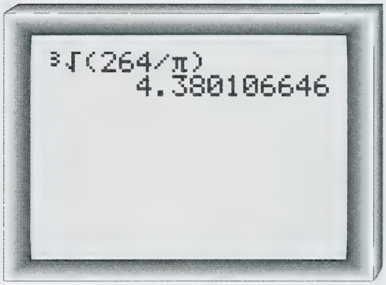
$264 = \pi d^3$

$\frac{264}{\pi} = d^3$

$d = \sqrt[3]{\frac{264}{\pi}}$

$\doteq 4.4$

The new diameter is about 4.4 cm.



```
³√(264/π)
             4.380106646
```

Note: Do not clear the calculator. Use the answer key in exercise 7.b.

b. Calculate the new surface area.

$A = \pi d^2$

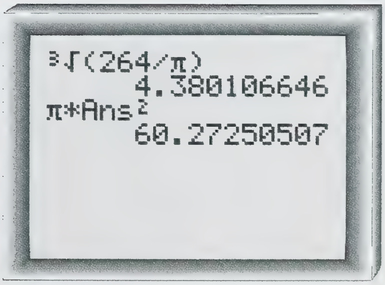
$\doteq \pi(4.4)^2$

$\doteq 60.3$

The new surface area is approximately 60.3 cm^2.

2nd [π] × 2nd [ANS]

x^2 ENTER



```
³√(264/π)
             4.380106646
π*Ans²
             60.27250507
```

Activity 5 (continued)

4. Textbook question "Communicating the Ideas," p. 45

Answers will vary. Following is one sample answer.

In the equations (formulas) used to find the volume of a sphere and a cube, one of the variables is cubed. To find the value of this variable, given the volume, you must use the cube root. For example, the volume of a sphere is $V=\frac{1}{6}\pi d^3$ **or** $V=\frac{4}{3}\pi r^3$; to find the diameter or radius, given the volume, you must use the cube root.

The volume of a cube is $V=s^3$; to find the side, given the volume, you must use the cube root.

Activity 6: Ratio and Proportion

1. Methods will vary. One method is shown.

Step 1: Write a proportion.

$$\frac{\text{Drawn}}{\text{Actual}} \qquad \frac{1}{6400}=\frac{3}{x}$$

← The tower is 3 cm tall in the drawing.

Step 2: Calculate the actual height (in centimetres) of the Calgary Tower.

$$\frac{\text{Drawn}}{\text{Actual}} \qquad \begin{aligned} \frac{1}{6400}&=\frac{3}{x} \\ \frac{1}{6400}\times 6400x&=\frac{3}{x}\times 6400x \\ x&=19\,200 \end{aligned}$$

← Multiply each side by $6400x$.

The actual height is 19 200 cm.

Step 3: Calculate the actual height (in metres) of the Calgary Tower.

$$\begin{aligned} &100\ \text{cm}=1\ \text{m} \\ \therefore\ &19\,200\ \text{cm}=192\ \text{m} \end{aligned}$$

The actual height of the Calgary Tower is 192 m.

2. **Textbook exercise 1 of "Exercises: Checking Your Skills," p. 50**

1. **a.** **Step 1:** Calculate the height (in metres) of the tower in the drawing. That is, multiply the actual length by the scale factor of 0.0001.

$$0.0001 \times 300 = 0.03$$

The tower in the drawing is 0.03 m tall.

Step 2: Calculate the height (in centimetres) of the tower in the drawing.

$$1 \text{ m} = 100 \text{ cm}$$
$$\therefore\ 0.03 \text{ m} = 3 \text{ cm}$$

The tower in the drawing is 3 cm high.

b. **Step 1:** Convert 4 cm to metres.

$$100 \text{ cm} = 1 \text{ m}$$
$$\therefore\ 4 \text{ cm} = 0.04 \text{ m}$$

Step 2: Determine the scale factor.

$$\frac{\text{Drawn}}{\text{Actual}}$$

$$\frac{0.04}{300} = \frac{4}{30\,000}$$
$$= \frac{1}{7500}$$

The scale factor is $\frac{1}{7500}$.

3. **Textbook exercises 1, 2, and 3 of "Investigation 1: The Relationship Between Scale Factor and Area," pp. 46 and 47**

1.

Rectangle	Length (cm)	Width (cm)	Perimeter (cm)	Area (cm²)
A	1	3	8	3
B	2	6	16	12
C	5	15	40	75

Activity 6 (continued)

Calculations

$$P = 2\ell + 2w = 2(1) + 2(3) = 2 + 6 = 8$$

The perimeter is 8 cm.

$$P = 2\ell + 2w = 2(2) + 2(6) = 4 + 12 = 16$$

The perimeter is 16 cm.

$$P = 2\ell + 2w = 2(5) + 2(15) = 10 + 30 = 40$$

The perimeter is 40 cm.

Remember to sketch each rectangle on grid paper.

$$A = \ell w = 1 \times 3 = 3$$

The area is 3 cm^2.

$$A = \ell w = 2 \times 6 = 12$$

The area is 12 cm^2.

$$A = \ell w = 5 \times 15 = 75$$

The area is 75 cm^2.

2.

Rectangle	Scale Factor	Ratio of Perimeters	Ratio of Areas
B to A	$2:1=2$	$16:8=2$	$12:3=4$
C to A	$5:1=5$	$40:8=5$	$75:3=25$
C to B	$5:2=\frac{5}{2}$	$40:16=\frac{40}{16}$ $=\frac{5}{2}$	$75:12=\frac{75}{12}$ $=\frac{25}{4}$

3. **a.** The ratio of the perimeters is equal to the scale factor.

b. The ratio of the areas is equal to the square of the scale factor.

$$2^2 = 4 \qquad 5^2 = 25 \qquad \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

4. Textbook exercise 6 of "Exercises: Checking Your Skills," p. 52

6. **a.** Doubling the sides of a rectangle increases the area by a factor of 4.

$2^2 = 4$

b. Step 1: Find the new area of the rectangle.

$$10 + 20 = 30$$

The new area is 30 cm^2.

Step 2: Find the scale factor.

$$\frac{\text{new area}}{\text{original area}} \quad \frac{30}{10} = \frac{3}{1}$$

The scale factor is 3.

Step 3: Express the scale factor as a decimal rounded to 3 decimal places.

$$\sqrt{3} \doteq 1.732$$

Each dimension was multiplied by a factor of approximately 1.732.

5. Textbook exercises 1 to 4 of "Investigation 2: The Relationship Between Scale Factor, Surface Area, and Volume," p. 47

1.

Sphere	Radius (cm)	Surface Area (cm^2)	Volume (cm^3)
A	1	12.57	4.19
B	3	113.10	113.10
C	5	314.16	523.60

Calculations

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi(1)^2 \\ &\doteq 12.57 \end{aligned}$$

The area is about 12.57 cm^2.

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi(3)^2 \\ &\doteq 113.10 \end{aligned}$$

The area is about 113.10 cm^2.

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi(5)^2 \\ &\doteq 314.16 \end{aligned}$$

The area is about 314.16 cm^2.

Activity 6 (continued)

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi (1)^3$$
$$\doteq 4.19$$

The volume is about 4.19 cm^3.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi (3)^3$$
$$\doteq 113.10$$

The volume is about 113.10 cm^3.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi (5)^3$$
$$\doteq 523.60$$

The volume is about 523.60 cm^3.

2.

Sphere	Scale Factor	Ratio of Surface Areas	Ratio of Volumes
B to A	$3:1=3$	9	27
C to A	$5:1=5$	25	125
C to B	$5:3=\frac{5}{3}$	$\frac{25}{9}$	$\frac{125}{27}$

Ratio of Surface Areas

$$\text{B to A } = \frac{\cancel{4}\cancel{\pi}(3)^2}{\cancel{4}\cancel{\pi}(1)^2}$$
$$= \frac{9}{1}$$
$$= 9$$

$$\text{C to A } = \frac{\cancel{4}\cancel{\pi}(5)^2}{\cancel{4}\cancel{\pi}(1)^2}$$
$$= \frac{25}{1}$$
$$= 25$$

$$\text{C to B } = \frac{\cancel{4}\cancel{\pi}(5)^2}{\cancel{4}\cancel{\pi}(3)^2}$$
$$= \frac{25}{9}$$

Ratios of Volumes

$$\text{B to A } = \frac{\cancel{\frac{4}{3}}\cancel{\pi}(3)^3}{\cancel{\frac{4}{3}}\cancel{\pi}(1)^3}$$
$$= \frac{27}{1}$$
$$= 27$$

$$\text{C to A } = \frac{\cancel{\frac{4}{3}}\cancel{\pi}(5)^3}{\cancel{\frac{4}{3}}\cancel{\pi}(1)^3}$$
$$= \frac{125}{1}$$
$$= 125$$

$$\text{C to B } = \frac{\cancel{\frac{4}{3}}\cancel{\pi}(5)^3}{\cancel{\frac{4}{3}}\cancel{\pi}(3)^3}$$
$$= \frac{125}{27}$$

3. **a.** The ratio of the surface areas of two spheres is equal to the square of the scale factor.

$$\begin{aligned}\text{B to A} &= 3^2 \\ &= 9\end{aligned} \qquad \begin{aligned}\text{C to A} &= 5^2 \\ &= \frac{25}{9}\end{aligned} \qquad \begin{aligned}\text{C to B} &= \left(\frac{5}{3}\right)^2 \\ &= \frac{25}{9}\end{aligned}$$

b. The ratio of the volumes of two spheres is equal to the cube of the scale factors.

$$\begin{aligned}\text{B to A} &= 3^3 \\ &= 27\end{aligned} \qquad \begin{aligned}\text{C to A} &= 5^3 \\ &= 125\end{aligned} \qquad \begin{aligned}\text{C to B} &= \left(\frac{5}{3}\right)^3 \\ &= \frac{125}{27}\end{aligned}$$

4. **a.** The scale factor of this sphere to sphere B in exercise 1 is 2. **Note:** $\frac{6}{3} = 2$

The surface area of this sphere is equal to the square of the scale factor; it will be 4 times the surface area of sphere B. **Note:** $2^2 = 4$

$$4 \times 113.10 = 452.40$$

The surface area of this sphere is about $452.40\ \text{cm}^2$.

b. The scale factor of the volume of this sphere to sphere B in exercise 1 is 2.

The volume of this sphere is equal to the cube of the scale factor; it will be 8 times the volume of sphere B. **Note:** $2^3 = 8$

$$8 \times 113.10 = 904.8$$

The volume of this sphere will be about $904.8\ \text{cm}^3$.

6. **Textbook exercise 2 of "Exercises: Checking Your Skills," p. 50**

2. Because the scale factor is $\frac{1}{4}$, the volume of the Moon will be $\frac{1}{64}$ the volume of Earth.

Note: $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

Activity 6 (continued)

7. Textbook exercise 1 of Part A of "What Should I Be Able To Do?" p. 57

1. Answers will vary. In the following sample answers, a baseball was used for the sphere representing the sled dog curled up in a ball.

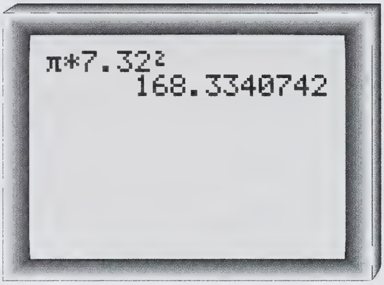
   a. A vernier caliper is used to measure the diameter of the ball because a vernier caliper is designed to take outside measurements and is precise to 0.01 cm.

   The diameter of the baseball is 7.32 cm.

   b.

$$
\begin{aligned}
A &= \pi d^2 \\
&= \pi (7.32)^2 \\
&\doteq 168.3
\end{aligned}
$$

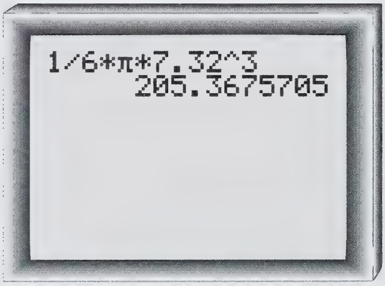
   The surface area of the baseball is about 168.3 cm^2.



```
π*7.32²
          168.3340742
```

$$
\begin{aligned}
V &= \frac{1}{6}\pi d^3 \\
&= \frac{1}{6}\pi (7.32)^3 \\
&\doteq 205.4
\end{aligned}
$$

   The volume of the baseball is approximately 205.4 cm^3.



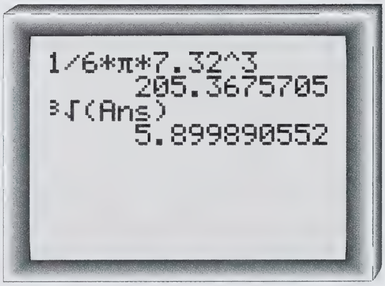
```
1/6*π*7.32^3
          205.3675705
```

   c.

$$
\begin{aligned}
V &= s^3 \\
\sqrt[3]{V} &= s \\
s &\doteq 5.9
\end{aligned}
$$

V is equal to the answer calculated in part b.

   If a cube has the same volume as the baseball, each side of the cube would be about 5.9 cm.



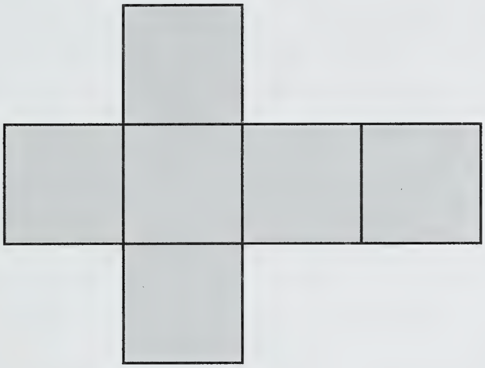
```
1/6*π*7.32^3
          205.3675705
³√(Ans)
          5.899890552
```

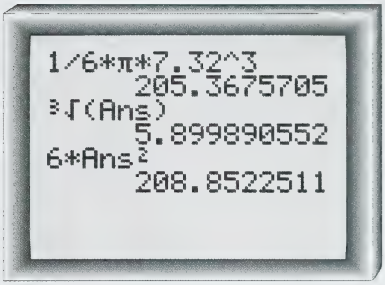
d. $A = 6s^2$

$\doteq 208.9$

The length of each side is equal to the answer calculated in part c.

The surface area of a cube with the same volume as the baseball is approximately 208.9 cm^2.



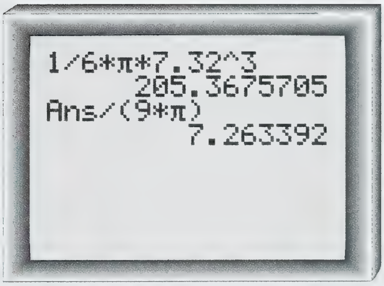


```
1/6*π*7.32^3
        205.3675705
³√(Ans)
        5.899890552
6*Ans²
        208.8522511
```

e.

$$V = \pi r^2 h$$
$$V = \pi (3)^2 \; h$$
$$V = 9\pi h$$
$$\frac{V}{9\pi} = h$$
$$h \doteq 7.3$$

$$V = \frac{1}{6}\pi d^3$$
$$= \frac{1}{6}\pi (7.32)^3$$



```
1/6*π*7.32^3
        205.3675705
Ans/(9*π)
           7.263392
```

If a cylinder has the same volume as the sphere (baseball) and a radius of 3 cm, its height will be about 7.3 cm.

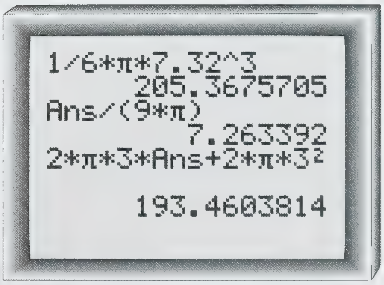
Note: You need to recalculate the volume in 1.b. and keep the calculator running.

f. $A = 2\pi rh + 2\pi r^2$

$= 2\pi (3)\, h + 2\pi (3)^2$

$\doteq 193.5$

If a cylinder has the same volume as the sphere, its surface area will be about 193.5 cm^2.



```
1/6*π*7.32^3
        205.3675705
Ans/(9*π)
           7.263392
2*π*3*Ans+2*π*3²

        193.4603814
```

g. The sphere has a surface area of about 168.3 cm^2. The cylinder has a surface area of about 193.5 cm^2.

$$168.3 < 193.5$$

Therefore, the sphere has the smaller surface area.

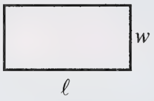
h. The surface area of a sphere is less than the surface area of a cylinder or a cube with the same volume.

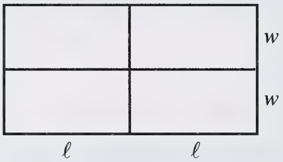
$$168.3 < 193.5 < 208.9$$

Because a mammal loses heat in proportion to its surface area, a sled dog would lose less heat if it were curled in a ball (sphere).

8. Textbook exercise 1 of "Discussing the Ideas," p. 50

1.





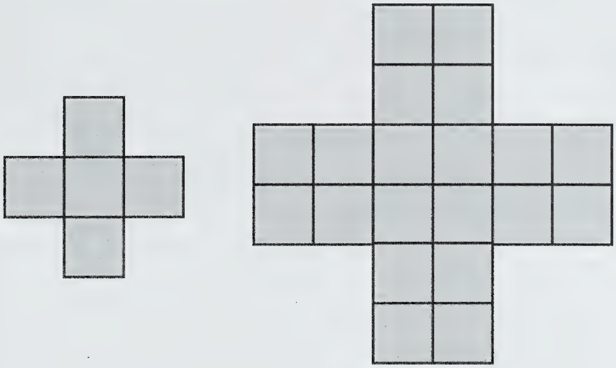
$$A = \ell w$$

$$\begin{aligned} A &= (2\ell)(2w) \\ &= 4\ell w \end{aligned}$$

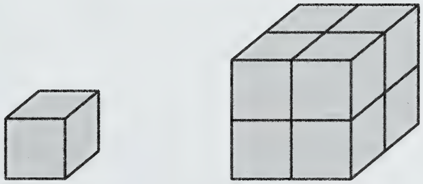
When the dimensions of a rectangle are doubled, the area of the rectangle is not doubled. Instead, the area is quadrupled.

9. Textbook question "Communicating the Ideas," p. 53

When the dimensions of a box are doubled, the surface area of the box is quadrupled.



When the dimensions of a box are doubled, the volume of the box is increased by a factor of 8.



Follow-up Activities

1. Textbook exercises 2 and 3 of Part B of "What Should I Be Able To Do?" pp. 57 and 58

2. **a.** vernier caliper or micrometer
b. tape measure or trundle wheel
c. vernier caliper
d. micrometer

3. In order to compare the measurement for precision and accuracy, first convert 21.23 mm to centimetres.

$1 \text{ mm} = 0.1 \text{ cm}$

$\therefore \; 21.23 \text{ mm} = 2.123 \text{ cm}$

a. Compare the measurements for precision:

2.12 cm ← nearest 0.01 cm
2.123 cm ← nearest 0.001 cm
21.2 cm ← nearest 0.1 cm

The second measurement is most precise.

b. The first and second measurements, 2.12 cm and 21.23 mm, are very close in size because 21.23 mm = 2.123 cm. The measurement 21.2 cm is much greater in size. The third measurement must be inaccurate because the same object is being measured.

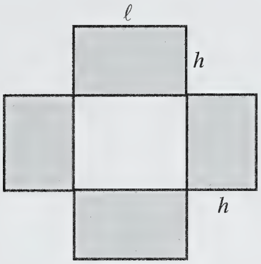
c. The measurement 2.12 cm was taken with a vernier caliper because a vernier caliper can measure the length of objects to the nearest 0.01 cm.

The measurement 21.23 mm was taken with a micrometer because a micrometer can measure the length of objects to the nearest 0.01 mm.

The measurement 21.2 cm was taken with a ruler because a ruler can measure the length of objects to the nearest 0.1 cm.

2. **Textbook exercises 5 and 6 of Part B of "What Should I Be Able To Do?" p. 58**

5. **Step 1**: Find the surface area of all the walls. It may help to visualize the room as a net.



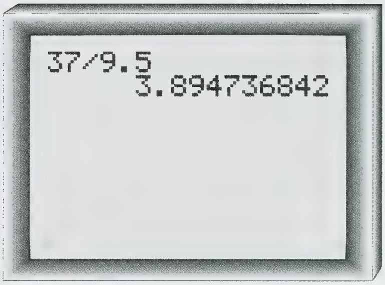
$$\begin{aligned} A &= 2\ell h + 2wh \\ &= 2(4\times 2.5) + 2(3.4\times 2.5) \\ &= 20 + 17 \\ &= 37 \end{aligned}$$

or

$$\begin{aligned} A &= Ph \\ &= (2\ell + 2w)h \\ &= (2\times 4 + 2\times 3.4)\,2.5 \\ &= (8 + 6.8)\,2.5 \\ &= (14.8)(2.5) \\ &= 37 \end{aligned}$$

The area of the walls is 37 m^2.

Step 2: Find the number of cans of paint needed.



```
37/9.5
          3.894736842
```

Note: Because Angey cannot buy a partial can of paint, round up the answer to the nearest whole number.

$37 \div 9.5 \doteq 4$

Angey must buy 4 cans of paint.

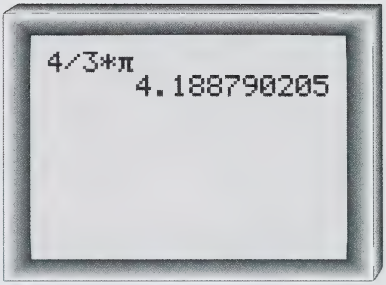
Step 3: Find the cost of the paint.

$4 \times 7.99 = 31.96$

The paint will cost \$31.96.

Follow-up Activities (continued)

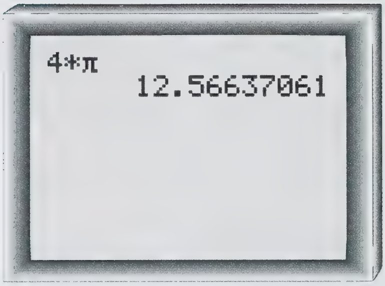
6. **a.** $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi (1)^3 \\ &= \frac{4}{3}\pi \\ &\doteq 4 \end{aligned}$$



```
4/3*π
          4.188790205
```

The volume of the ball is about 4 cubic feet.

b. $$\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi (1)^2 \\ &= 4\pi \\ &\doteq 13 \end{aligned}$$



```
4*π
          12.56637061
```

The surface area of the ball is approximately 13 square feet.

Extra Help

1.

Feet	3	$1.\overline{6}$	4	$5.8\overline{3}$
Inches	36	20	48	70

2.

Yards	8	$9.\overline{3}$	14	$5.\overline{6}$
Feet	24	28	42	17

Enrichment

1. From the table, you see that 1 mile $\doteq 1.609$ km.

1500 m = 1.5 km

The distance of 1500 m is shorter than 1 mile because 1.5 km is less than 1.609 km.

2. **Textbook exercises 2, 3, and 4.b. of “Exercises: Checking Your Ideas,” p. 20**

   2. $23 \text{ feet } 9 \text{ inches} = 285 \text{ inches}$

      $285 \text{ inches} \doteq 7.24 \text{ m}$

      The three-point field goal line is approximately 7.24 m from the centre of the basket.

   3. $100 \text{ yards} \doteq 91.44 \text{ m}$

      The 100-m dash is greater than the 100-yard dash because 100 m is greater than 91.44 m.

      $100 - 91.44 \doteq 8.56$

      The 100-m dash is greater by approximately 8.56 m (or 9.36 yards).

   4. **b.** $5\,200\,000 \text{ square feet} = 483\,080.00 \text{ m}^2$

      The mall covers $483\,080.00 \text{ m}^2$.

3. **Textbook exercises 1 to 3 of “Investigation 2: Using Technology,” pp. 16 and 17**

   1. 5 miles = 8800 yards

   2.

Miles	Yards
5	8880
10	17 600
15	26 400
20	35 200
25	44 000
30	52 800

   3. The calculator multiplies the number of miles by 1760 to find the numer of yards.

4. The measurement conversions were made using rounding. In some cases, units were rounded up; in other cases, units were rounded down. This way, measurements were numbers that were easy to work with (for example, multiples of 5 or 10). **Note:** Rounding did not cause significant differences in the quantities.

Module Project: The Dream Room

Completing the Project

1. **Textbook exercise 10 of Part C of "What Should I Be Able To Do?" p. 59**

10. Responses will vary. A sample response is given.

The student has made a very good floor plan of his or her dream room. The scale of the floor plan is indicated at the top of the drawing. All the pieces of furniture and the bathroom fixtures (toilet, sink, and shower) are clearly labelled. However, it is not necessary to indicate the dimensions and area of the bed, dresser, desk, sink, toilet, and shower. As well, the area of the toilet seems to be miscalculated. The area of the toilet, which is labelled as 14 square feet, is clearly less than the area of the dresser, which is labelled as 8 square feet; this does not make sense. The bathroom cabinet size is not clear or accurate. The student has made several other errors in calculations:

- The shower area should be one-quarter of the area shown—4.9 square feet.
- The area of the chair has been calculated using the diameter (of the chair seat) squared; the radius should be squared $\left(d = 2,\ r = 1;\ A = 3.14 \text{ square feet}\right)$.

The student has included most of the measurements needed for the floor coverings. In addition, the student has shown the calculations of the area of the floor. However, the student ignored the depths of the doorways; an additional 4.5 square feet should have been added for the floor in the three doorways, because each doorway is 3 feet by 0.5 feet.

The student did not subtract the area of the bathroom fixtures from the total bathroom floor area, and this was the right decision as there will be some wastage.

The student has included most of the dimensions needed to plan for painting or wallpapering the walls. In addition, the student has included the calculation of the area of the walls. However, the student did not clearly indicate the height of the walls, windows, and doors on the floor plan. On the other hand, stating that the windows are 2 feet above the flooring was not necessary. In the calculation of the area of the walls, the student did not make allowances for doors and windows in the calculations of the area of the walls. In addition, the student did not indicate how the area of the windows and doors were calculated to be 65 square feet. The ceiling area was not mentioned at all. The area of window and door casings, baseboards, and doors was not mentioned.

The student has not included a night stand. The foot of the bed is too close to the dresser (it would be difficult to walk through this small space and difficult to fully open dresser drawers).

Note: The student mentions an extension in the cost analysis sheet. Nothing was shown in the drawing to include such a feature.

2. Textbook exercise 11 of Part C of "What Should I Be Able To Do?" p. 60

11. Responses will vary. A sample response is given.

Nothing has been left out of the room; however, it was not necessary to include the cost of building an extension or the cost of new bathroom fixtures. The student could have assumed the room and fixtures already existed.

The cost of the linoleum seems low. It is not likely that you would buy linoleum at $0.79 per square foot. The cost of window treatment seems low; two sets of drapes or blinds would cost more than $11.99. A desk and chair would probably cost more than $47; two sets of shelving would probably cost more than $45. The student should have included the cost of a night stand.

As mentioned previously in the response to exercise 10, the student made errors calculating the area of the floor to be covered and the area of the walls to be painted. The cost of paint is on the low side. The cost of the extension is vastly underestimated and the extension is not included in the floor plan. This extension would increase the room size by 105 square feet.

The error in calculating the cost of flooring may have been a conversion error, as linoleum and carpet is often sold by the square yard.

Note: Many of the costing errors may have been made by simply making choices on looks and price. This is why it is important to talk to experts in the field. They can help direct you to the types of products required. It is important to consider function in your choices. For example, paint used in a bathroom should be scrubbable and wallpaper should be vinyl. Functions like these add to the cost of the product you may choose.

Note, too, that the student has included federal and provincial sales taxes. Your calculations will reflect the costs incurred in your province.

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